PRESENCE OF SITUATIONAL AND MATHEMATICAL MODELS IN MEXICAN MATHEMATICS TEXTBOOKS FOR MIDDLE SCHOOL: AN INITIAL CATEGORIZATION AND QUANTIFICATION

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Abstract

Problem solving and, since recently, mathematical modeling are of a crucial importance in processes of learning and teaching school mathematics. Both processes are greatly influenced by the ways mathematics textbooks treat problem solving and mathematical modeling. Main objective of this documental research was to determinate basic features and quantify presence of situational and mathematical models in 480 problem formulations proposed by authors of Mexican of 29 mathematics textbooks for middle school (IX grade). The results show that students are not informed about (1) the differences between situational and mathematical models and (2) their role and importance in learning and practice of mathematical modeling. In addition, in many examples, complete or incomplete mathematical models are inserted into situational models (drawing or photograph). Finally, three selected problems with different visual complements were given to 43 students in order to explore which type of drawings (situational, mathematical or mixed) students carry out when asked to make them. Students' performances were analyzed and compared with the common images (drawings and/or photographs) provided by mathematics textbooks to complement problems formulations.

Keywords: Situational model, mathematical model, mathematical modeling, mathematics textbooks

INTRODUCTION

To promote deeper learning and 21st century skills, educators have to design and implement successfully those learning sequences that foster a cluster of cognitive competences: critical thinking, non-routine problem solving, and constructing and evaluating evidence-based arguments (Pellegrino & Hilton, 2013).

Solving problems in mathematics education had always two related roles. One was to give students opportunities to learn mathematics through problem solving and the other one was to use mathematical problems as an evaluation tool to measure students' learning progresses in formative and summative assessments.

As many students weren't successful in learning to solve problems, there was a big need to give them a few general ideas and steps that can be used strategically in solving almost any problem in school mathematics. The first systematic approach to solving mathematical problems, with a clear teaching purpose on mind, was given by Gerge Polya in his famous book "*How to solve it. A new aspect of mathematical method*" (Polya, 1945). Polya's ideas were later elaborated, extended and popularized by Schoenfeld (1985) and became main focus of world



mathematics education in the last 30 years. As it is well known, the four basic steps in Polya's approach are (1) Understand problem; (2) Devise a solution plan; (3) Carry out the plan and (4) Look back to analyze and check the solution (Polya, 1945). Polya and Schoenfeld strongly suggested "make a drawing" as a useful heuristic in analyzing and exploring problem more carefully. Nevertheless, many students need an instructional support in practicing and acquiring such a "strategic routine" in solving mathematical problems (Essen & Hamaker, 1990; van Dijk et all., 2003; Nunokawa, 2004).

A very important research result has shown that characteristics of students' spontaneously generated visual-spatial representations of problem situation are highly correlated with students' performance in solving problem (Hegarty & Kozhevnikov, 1999). In general, there are two types of visual-spatial representations: (1) *schematic representations* encode the spatial relations described in a problem and *pictorial representations* encode the visual appearance of the objects described in the problem. Students who use schematic spatial representations are likely to be successful in mathematical problem solving, while those who generated pictorial representations have much less success.

THEMATIC FRAMEWORKS OF THE RESEARCH: MATHEMATICAL MODELING IN MATHEMATICS EDUCATION AND TEXTBOOKS

Social and pedagogical demands to have more "applied problems" in school mathematics (application of mathematics in real problem situations called "contexts") has created new challenges for mathematics learning and teaching through problem solving (Blum & Niss, 1991). Real-world context or problem situation should be first understood and then simplified by appropriate mathematical consideration and assumptions to obtain a mathematical model that can be used for planning a solution. In other words, a problem solver should work out a three-part entity SMR: a real problem situation (S); a collection of mathematical entities (M) and relations (R) by which objects and relations of S are related to objects and relations of M.

This non-trivial transition between "situation models" and "mathematical models" is the most difficult part of "mathematical modeling" (**Figure 1**) that became recently an important trend in mathematics education, complementing previous problem solving approach in the domain of contextualized mathematical problems (Blum et all., 2007; Stillman et all., 2013)



Figure 1. Modeling Cycle



In the first step "Understanding", problem solver hast to construct a correct mental model of the situation, called "*situational model*". A common way to represent it is by making a drawing. The second step "Simplifying/Structuring" consists of selecting relevant data, making simplifying assumptions and structuring situation in order to get a "*real model*". In the third step "Mathematising", mathematical concepts and relationships are used to get a "*mathematical model*" of the situation that will be used for planning a mathematical solution. Borromeo Ferri has studied and described fine psychological and cognitive aspects of different transition paths between "situation model" and "mathematical model" (Borromeo Ferri, 2006).

Taking into account many possible meaning of the term "model", its use in students' modeling tasks might be unproductive. In that sense, the term "drawing", that is student-friendly, can be more helpful. Rellensmann, Schukajlow and Leopold (2017) took that opportunity and considered the role and importance of "situational drawings" and "mathematical drawings" in mathematical modeling problems. They define y specify these two type of drawing as follows:

Situational drawing "is an externalized representation of the model of situation that pictorially depicts the objects described in the problem situation according to their visual appearance. Therefore, a situational drawing is a drawing with a low level of abstraction."

"The situational drawing depicts the task's relevant objects pictorially: a fire engine with wheels, an extended ladder with clearly visible rungs, and a house with a pitched roof."

Mathematical drawing "is an abstract drawing because it provides an externalized representation of the mathematical model. A mathematical drawing depicts only solution-relevant objects from the problem situation, and these are reduced to their relevant mathematical features."

"In the mathematical drawing, all objects are reduced to their relevant mathematical features: The fire engine, the house, and the ladder are reduced to line segments, and their heights or length are noted in the drawing."

Being aware (through their own research) of students' lack of "strategic drawing knowledge", these authors propose a drawing-oriented modeling instruction:

"In order to enhance students' ability to use drawings efficiently, classroom practice should strengthen students' strategic knowledge about drawing and their competency to create drawings of high accuracy. If students know what a helpful drawing is and how to construct and use such a drawing, they are more likely to generate appropriate, highaccuracy drawings, and this will increase their modelling performance down the line. Students' drawing skills can be improved by providing them with instructional support and sufficient practice in drawing (Rellensmann, Schukajlow and Leopold, 2017)".

It is known (Valverde et all., 2002) that classroom practice ("implemented curriculum") is more dependent on the content and quality of textbooks ("potentially implemented curriculum") than on official programs (intended curriculum). Taking into account increased curricular presence of mathematical modeling, it is surprising that there are only a few initial and limited studies that paid attention to how mathematical modeling is treated in textbooks. The results of such studies in Netherland (Zwaneveld et all., 2017) and in United States of America (Meyer, 2015) show clearly that features and structure of modeling tasks are unsatisfactory and unlikely to give students an opportunity to practice and learn how to model real-world situation mathematically. Although in Mexican curriculum for primary school mathematical modeling isn't



explicitly stated as a learning goal, there was a claim that textbooks should be modified in order to have more real modeling tasks (Quiroz & Rodríguez, 2015).

AIM AND METHODOLOGY OF THE RESEARCH

The aim of this documental research (Delamont, 2013) was to initially explore, categorize and quantify the presence and type images that complement problem formulations in Mexican mathematics textbooks for third grade of middle school (in Spanish "tercer grado de secundaria", corresponding to ninth year of schooling). Our special focus was to determine whether these images have features of situational and/or mathematical models that might help students better solve contextualized problems.

With that emphasis, our research questions were:

How present are situational and mathematical models in Mexican mathematics textbooks for third grade of middle school?

How such a treatment of situational and mathematical models might help or mislead students in their struggle to understand, practice and learn basic processes in mathematical modeling?

The sample for this documental research was formed by 29 mathematics textbooks, officially authorized for their use in Mexican public schools. The list of analyzed textbooks is given in Appendix.

THE RESULTS OF TEXTBOOK ANALYSIS

We found 780 mathematical problems in which students' construction of situational and mathematical models would be useful in finding solutions. In 41 problems, that were formulated without an image, students are not asked to provide either situational or mathematical drawings. In 3 problems students are given only a decorative image (photograph) that can't help them plan and find a solution. 17 problems were supplemented with two separated images. The first image (photograph or drawing) describe visually problem situation and might be considered as some kind of "mathematical model". The second represents a mathematical drawing. One example of this type of visual complement is given in the *Box 1*.

Guadalupe projects a shadow by placing herself next to the post of a sun clock. If you want to know the distance from the tip of hear head to the point where the shadow ends, what calculations should you make? Consider that Guadalupe measures 1.55 m and the post is 30 cm high.

Problem Formulation

Look at the lines on the right that indicate the length of the shadow projected by the clock and solve.



Box 1. An example of two-image complement of a mathematical problem



Nevertheless, students are not informed why two images are provided and how the second one is related to and derived from the first one. Resting 419 problem formulations were complemented by a single image (drawing or photograph). According their connections with situational and mathematical models, these images can be divided in four basic categories. Three of these categories are further specified by their division into subcategories.

Category 1: Situational model and complete mathematical model are presented together

The image provided has concrete pictorial elements, corresponding to problem situation. A simplified schematic drawing, corresponding to a complete mathematical model, is inserted in the image without any comment. In almost all cases, "complete mathematical model" was a corresponding triangle (with all three sides clearly drawn).

Students are not informed why pictorial and schematic parts of the image are presented in that way. According to the character of pictorial part of the image, examples in this category can be divided into two subcategories.

Images in **subcategory 1A** have a complete mathematical drawing (triangle) inserted in situational drawing (*Box 2a*).

Problem formulation

A lighthouse is on the beach and is 675 m high. From the top of it and in an angle of depression of 76^{0} a certain boat can be seen, as shown in the figure ... How far from the lighthouse base is the boat?



Box 2a. Single image containing a complete mathematical drawing inserted into a situational drawing

Images in **subcategory 1B**, contain a complete mathematical drawing (triangle) inserted into a situational photograph (*Box 2b*).

Problem formulationSituational photograph with complete
mathematical drawingHans, a German tourist, visited the Zocalo of Mexico City
and observed that the shadow projected by the flagpole of
the Mexican flag is approximately 20 m long. He looked
in his portable navigator and read that this national
emblem measures 50 m in height. Then he took a photo,
traced the ABC triangle over it and sent it to his
mathematical friends he has in Berlin, to show them the
flag and challenge them to calculate the distance from the
tip of the flagpole to the final end of its shadow. How
much does this distance measure?Situational photograph with complete
mathematical drawing

Box 2b. Single image containing a complete mathematical drawing inserted into a situational photograph



Category 2: Situational model and incomplete mathematical model are presented together

As in previous category, the image provided has concrete pictorial elements, corresponding to problem situation, but instead of complete mathematical model an incomplete triangle is inserted (two or only one relevant side is clearly drawn). Again, authors don't find it necessary to inform students why pictorial and schematic parts of the image are presented in that way. According to the character of pictorial part of the image, examples in this category can be divided into two subcategories.

Images in subcategory 2A have an incomplete triangle inserted into a situational drawing (Box 3a).



Box 3a. Single image containing an incomplete mathematical drawing inserted into a situational drawing

Images in **subcategory 2B** have an incomplete triangle inserted into a situational photograph (*Box 3b*).



Box 3a. Single image containing an incomplete mathematical drawing inserted into a situational photograph

Category 3: Only situational model without any or with some quantitative data

The images in this category do not provide mathematical drawings (models) that might help students plan their mathematical solutions. According to the type of visual information related to the problem situation (drawing or photograph) and the presence of quantitative data (no data o some data), four subcategories were identified.



Subcategory 3A contains situational drawings without quantitative data (Box 4a).



Box 4a. Single image containing a situational drawing without quantitative data

Situational photographs without quantitative data belong to subcategory 3B (Box 4b).



Box 4b. Single image containing a situational photograph without quantitative data

Situational drawings with some quantitative data fall into subcategory 3C (Box 4c).



Box 4c. Single image containing a situational drawing with some quantitative data

Subcategory 3D describes those images in which quantitative data are inserted into situational photographs (Box 4d).



Box 4c. Single image containing a situational photograph with some quantitative data



Category 4: Only complete mathematical model

These images contain only complete mathematical models without mentioning how schematic drawings are related to and which simplifying assumptions were necessary to derive them from situational information given in problem formulation (*Box 5*).



Box 5. Single image containing only mathematical model

Numerical quantification of presence of described categories and subcategories is given in Table 1.

Table 1.	Categories of di	ifferent combination	on of situationa	l and mathematica	l models in single image	
			annroach			

Category (N of examples in a category)	Short description of a subcategory or a category	Number of examples in a subcategory or category	
Category 1 Situational model and	Subcategory 1A Situational model (in form of a drawing) a complete mathematical model are "mixed" in a single image	nd 71	
complete mathematical model are presented together (90 examples)	Subcategory 1B Mathematical model (in form of a photograph) and complete mathematical model are "mixed" a single image.	'in 19	
Category 2 Situational model and	Subcategory 2A Situational model (in form of a drawing) a incomplete mathematical model are "mixed" in a single ima	ind 91	
incomplete mathematical model are presented together (100 examples)	Subcategory 2B Mathematical model (in form of a photograph) and incomplete mathematical model are "mixed in a single image.	d" 9	
	Subcategory 3A Only situational model (in form of a drawi with no quantitative data is presented.	ing) 31	
Category 3 Only situational model without or with some	Subcategory 3B Only situational model (in form of a photograph) with no quantitative data is presented.	11	
quantitative data (146 examples)	Subcategory 3C Only situational model (in form of a drawn with some quantitative data is presented.	ing) 87	
	Subcategory 3D Only situational model (in form of a photograph) with some quantitative data is presented.	17	
Category 4 Only complete mathematical model (83 examples)	Category 4 Only complete mathematical model is presented without any further comment or clarification.	d 83	
		419	

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Now we are in position to answer our research questions. The first one was: How present are situational and mathematical models in Mexican mathematics textbooks for third grade of middle school?

Taken together, presence of two first categories show that the most common approach of textbook authors to mathematical modeling is to insert a complete or an incomplete mathematical model into a visual representation related to the problem situation. It happens in 45 % of analyzed mathematical problems.

Second common approach is to present only situational model (drawings or photograph) without or with some quantitative data. It was found in almost 35 % of complementing images. Less common approach (almost 20 %) is to complement problem formulation only with a mathematical drawing.

The second research question was: How such a treatment of situational and mathematical models might help or mislead students in their struggle to understand, practice and learn basic processes in mathematical modeling?

Absence of definitions of situational and mathematical models along with indication about their different role in mathematical modeling, mixing situational and mathematical models, and presenting only situational o mathematical models in arbitrary ways will likely confuse students, leaving them in adverse learning conditions. Students will be obliged (1) to guess reasons why visual complements to problem formulations have described features and (2) to improvise visualizations when having a task to produce a useful drawing for problem solving.

A SMALL-SCALE FOLLOW-UP STUDY

In order to explore initially how students' drawings, depend on visual complements and task indications, we selected three different problems and designed corresponding structured students' worksheets. Selected problems were:

1. Shadow problem

"A building projects a shadow 50 meters long when the angle of the Sun on the horizon is 70° . How tall is the building?"

This problem neither provides a visual complement nor asks students to make a drawing. In students' worksheet, the problem formulation was followed by three tasks:

a) To solve this problem, it is recommended to make a drawing. Draw yours in the space that follows.

b) Present calculations needed to get the solution.

c) The height of the building is _____ meters.

2. Drawbridge problem

"Each half of a drawbridge, as in the photo below, measures 130 m. If they reach a height of 50 m above the level of the road, what will be the separation between its two halves?"





This problem provides a visual complement (situational photograph above) but still doesn't ask students to make a mathematical drawing. In students' worksheet, the problem formulation was followed by three tasks:

a) To solve this problem, it is recommended to make a drawing. Draw yours in the space that follows.

b) Present calculations needed to get the solution.

c) The separation between two halves of the drawbridge is ______ meters.

3. Eagle problem

"An eagle fish hunter, flying at 800 m above sea level, discovers a fish in water surface at an angle of 30 degrees to the surface itself.



a) Draw the triangle that will be useful to find the distance between the eagle and the fish.

b) What is the trigonometric function that you will use to solve the problem?

c) Write the equation for the distance the fish and the eagle.

d) The distance between the fish and its predator is _____ meters."

This problem complements its situational description by a situational drawing in which an incomplete mathematical drawing is inserted. It is the only problem in which students are asked explicitly to make a "mathematical drawing" ("... the triangle that will be useful...).

The worksheets with these three problems were given to 43 students (29 students in ninth grade and 14 students in tenth grade). Categories of drawings generated by students and number of correct answers in each category are presented in the Table 2.

Problems (N of students' answers)	N of Situational drawings (N of correct answers in this category)	N of Mathematical drawings (N of correct answers in this category)	N of Mixed drawings (N of correct answers in this category)	Without a drawing (N of correct answers in this category)
Shadow (43)	7 (0)	7 (2)	29 (15)	0 (0)
Drawbridge (38)	6 (0)	26 (14)	5 (2)	1(1)
Eagle (39)	3 (0)	26 (2)	10 (0)	0 (0)

Table 2. Types of students' drawings and number of correct solutions

These preliminary results permit some tentative conclusions:

 (1) Absence of visual complement (shadow problem) leads majority of students to make a mixed drawing (mathematical drawing inserted into a situational drawing). This result shows that students are likely to imitate common textbook approach in their own drawing performance
(2) Presence of visual complement (drawbridge and eagle problem) increases the number of mathematical drawings.



The distribution of correct answers along different drawing performances is in resonance with known results (Hegarty & Kozhevnikov, 1999). In all problems, students who generate only a situational drawing are unable to get a correct result. In two problems (drawbridge and eagle problem), students who made a mathematical drawing had more correct answers. Nevertheless, in shadow problem, students with mixed drawing outperformed students with mathematical drawing. It is interesting to note that generating mathematical drawings doesn't lead necessarily to a correct answer. In drawbridge and eagle problems, the number of mathematical drawings is equal but numbers of correct answers differ greatly.

CONCLUSIONS

Treatment of mathematical modeling in Mexican mathematics textbooks for third grade of middle school isn't satisfactory:

Students are not informed by textbook authors about basic features of situational and mathematical models and their important role in mathematical modeling of real-world situations.

The dominant textbook approach is to mix situational and complete or incomplete mathematical models into a single image.

As this treatment isn't good for students' learning of mathematical modeling, many changes are needed for an improvement:

(1) Mathematical modeling should become an important curricular goal of Mexican mathematics education.

(2) Textbook treatment of modeling cycle should improve and clear differences between situational and mathematical models must be clearly stressed and supported by explicit examples.

(3) Courses on teaching mathematical modeling should be included in mathematics teachers' education.

APENDIX

The list of revised mathematics textbooks

01. Arriaga Robles, A. & Benítez, M. M. (2016). *Matemáticas 3 por Competencias*. México D.F.: Pearson.

02. Arriaga, A., Sesma, S. E., Pineda, V. H., Zavala, G., Compañ, M., Gutiérrez, J. J. & Zahoul, J. G. (2015). *Matemáticas 3. Matemáticamente Consolidación de Competencias*. México D.F.: Editorial Pearson.

03. Arteaga R. & Sánchez, A. (2017). *Trabajo en proceso 3. Matemáticas*. México D.F.: Oxford University Press.

04. Baltazar, C., Ruíz, E. & Ojeda, L. F. (2013). *Matemáticas 3. Explora*. México D.F.: Ediciones Castillo.

05. Castrejón, A., Vicuña A., Reyes, M. L. & Castrejón, O. S. (2016). *Retos Matemáticos 3*. México D.F.: Ediciones S.M.

06. Covián, E. (2015). Matemáticas 3. Ciudad de México: Editorial Nuevo México.

07. Cuevas, C. A., González, O. & Real, C. (2015). Matemáticas 3. México D.F.: Ríos de tinta.

08. De Icaza, A. (2014). Matemáticas 3. México D.F.: Editorial Santillana Secundaria.



09. Escareño F. & López, O. (2014). *Matemáticas 3: Desafíos matemáticos*. México D.F.: Editorial Trillas.

10. Farfán R. M., Cantoral R. A., Cabañas M. G., Ferrari, M. & Lezama, F. J. (2014). *Matemáticas 3*. México D.F.: Mc Graw Hill Education.

11. Filloy, E., Figueras, O., Ojeda, A.M., Rojano M. T. & Zubieta, G. (2014). *Matemáticas 3*. México D.F.: Editorial Esfinge.

12. Flores A.H, Gómez A. & Hernández A.E. (2014). *Convive con las Matemáticas 3*. México D.F.: Méndez Cortés Editores.

13. García, V., Villaseñor, R. & Montes, M.D. (2017). *Fortalezco mis Competencias Matemáticas 3*. México D.F.: Terra Esfinge.

14. García E. E., Villegas C. S., & López E. (2014). *Matemáticas 3*. México D.F.: Norma Ediciones.

15. Hernández J. M., Solano H. & Jiménez L. (2014). *Matemáticas 3. Estrategias de pensamiento*. México D.F.: Grupo Editorial Patria.

16. Mancera E. & Basurto E. (2014). Matemáticas 3. Serie Saberes México D.F.: Pearson.

17. Mata, D. & Nebbla, C. F. (2016). Fortalezco mis Competencias Matemáticas 3. México D.F.: Ediciones S.M.

18. Moreno, J.A. (2016). Retos Matemáticos 3. México D.F.: Ediciones S.M.

19. Núñez N.E., García D., Castillo R., & Tovar A. I. (2014). *Matemáticas 3*. México D.F.: Editorial Terracota.

20. Olea, A., Basurto, E. & Rivera M.L. (2014). *Matemáticas 3. Contexto matemático*. Ciudad de México: Norma Ediciones S.A de C.V.

21. Palmas S. A. & Balderas R. G. (2014). *Matemáticas 3*. México D.F.: Correo del Maestro.

22. Pérez M. & Pérez S., (2015). *Matemáticas 3: Construyo y aprendo matemáticas*. Ciudad de México: Fernández Editores.

23. Ramírez C. M., Castillo C. R. & Vergara R. G. (2017). *Matemáticas 3: Desafíos matemáticos*. México D.F.: Fernández Editores.

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