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NON-LINEAR DYNAMICS IN TUMORAL PROCESSES

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The dynamics of a laboratory grown cerebral tumor [1,2] cultivated from the celular stable line C6 of rat astrocyte glioma has been experimentally studied. The dynamic evolution of the interface profile is followed by digitalizing the images of two tumors at several times taken through a microscope, using a magnification of $40\times$ and $200\times$. In the following text these samples are refered to as T1 and T2 respectively. The growth conditions have been set to obtain a quasi-bidimensional pattern that grows radially. The fractal dimension D_f, a quantity that emerges in the study of disordered processes, has been demostrated to be useful when characterizing cells morphology [3]. A property related to D_f is the interface roughness. The tumor interface morphology has been characterized by D_f using the box-counting method and by the roughness exponent α [4]. The growth dynamics have been studied in terms of the temporal evolution of the mean radius, the fractal dimension and the roughness.

In general, the scaling properties of rough interface profiles are studied by means of the interface width function $W(L,t)$, with $W(L,t)^2 = \frac{1}{N}\sum_i^N [r_i(t) - < r >_L (t)]^2$, where r_i is the radial coordinate of the ith particle, L is the system size and N is the total number of particles [4]. However, in this work the range of variation of L is not large enough to do scaling analysis; therefore, the interface local width function $w(l,t)$ that measures the *rms* fluctuations of the interface over 'windows' of scale l is computed. To our knowledge, no work has been done on local width functions in circular symmetry. A method, that consists of computing the *rms* interface fluctuation about the *local average radius* (from the mass centre) in 'angular-sector-type windows' and averaging over all the windows is proposed[5].

The evolution of the mean radius of the tumor is shown in Fig. 1(a). Contrary to the expected exponential growth, the figure appears to show a linear behaviour. The fractal dimension measured by the box-counting method (Fig. 2(a) for T2) increases with time as can be seen in Fig. 2(b). Furthermore, the roughness exponent α decreases with time for both tumors (Fig. 1(b)) $0,55 \leq \alpha \leq 0,80$, in such a way that it satisfies the relation $D_f + \alpha = D_{\mathrm{Eucl}}$ [6]. It should be noted that a local width function can be obtain in the same way as a global width function: measuring the *rms* fluctuations about the average radius over subsets of l number of points (as in the linear case). The local roughness exponent α_{loc} can be computed by plotting this function against l [7,8]. However, the value of α obtained by stretching the interface and applying the standard local width function $w(l,t)$ for *linear* profiles is $\alpha = 0.86$, independently of both the magnification and the time. Furthermore, we have computed

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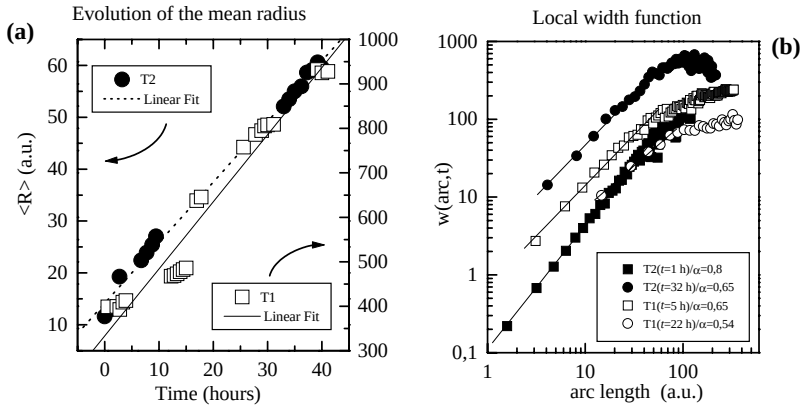

FIG.1. *(a) Time evolution of the tumors mean radii showing linear behaviour. (b) Roughness exponent computed with the 'angular' local width function.*

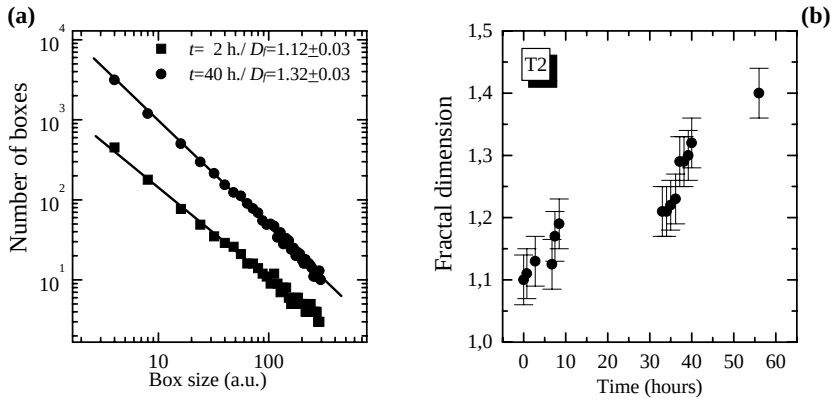

FIG.2. *(a) Computation of fractal dimension with box-counting for T2 Our method, taking angular sections on the interface. (b) Evolution of the fractal dimension for T2.*

the global width function $W(L,t)$ and estimated the *global roughness exponent* with the fifteen system sizes (corresponding to the different digitilized images) $\alpha_{\text{glob}} > 1$.

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