JORDAN CANONICAL FORM

11.52. Find all possible Jordan canonical forms for those matrices whose characteristic polynomial $\Delta(t)$ and minimum polynomial $m(t)$ are as follows:

(a) $\Delta(t) = (t-2)^4(t-3)^2$, $m(t) = (t-2)^2(t-3)^2$

(b) $\Delta(t) = (t-7)^5$, $m(t) = (t-7)^2$

(c) $\Delta(t) = (t-2)^7$, $m(t) = (t-2)^3$

(d) $\Delta(t) = (t-3)^4(t-5)^4$, $m(t) = (t-3)^2(t-5)^2$

11.53. Show that every complex matrix is similar to its transpose. (*Hint:* Use its Jordan canonical form and Problem 11.50.)

11.54. Show that all $n \times n$ complex matrices A for which $A^n = I$ but $A^k \neq I$ for $k < n$ are similar.

11.55. Suppose A is a complex matrix with only real eigenvalues. Show that A is similar to a matrix with only real entries.

CYCLIC SUBSPACES

• **11.56.** Suppose $T: V \to V$ is linear. Prove that $Z(v, T)$ is the intersection of all T-invariant subspaces containing v.

• **11.57.** Let $f(t)$ and $g(t)$ be the T-annihilators of u and v, respectively. Show that if $f(t)$ and $g(t)$ are relatively prime, then $f(t)g(t)$ is the T-annihilator of $u + v$.

11.58. Prove that $Z(u, T) = Z(v, T)$ if and only if $g(T)(u) = v$ where $g(t)$ is relatively prime to the T-annihilator of u.

11.59. Let $W = Z(v, T)$, and suppose the T-annihilator of v is $f(t)^n$ where $f(t)$ is a monic irreducible polynomial of degree d. Show that $f(T)^s(W)$ is a cyclic subspace generated by $f(T)^s(v)$ and it has dimension $d(n-s)$ if $n > s$ and dimension 0 if $n \leq s$.

RATIONAL CANONICAL FORM

11.60. Find all possible rational canonical forms for:

(a) 6×6 matrices with minimum polynomial $m(t) = (t^2+3)(t+1)^2$

(b) 6×6 matrices with minimum polynomial $m(t) = (t+1)^3$

(c) 8×8 matrices with minimum polynomial $m(t) = (t^2+2)^2(t+3)^2$

11.61. Let A be a 4×4 matrix with minimum polynomial $m(t) = (t^2+1)(t^2-3)$. Find the rational canonical form for A if A is a matrix over (a) the rational field $\mathbf{Q}$, (b) the real field $\mathbf{R}$, (c) the complex field $\mathbf{C}$.

11.62. Find the rational canonical form for the Jordan block $\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}$.

11.63. Prove that the characteristic polynomial of an operator $T: V \to V$ is a product of its elementary divisors.

11.64. Prove that two 3×3 matrices with the same minimum and characteristic polynomials are similar.