Quantum Tunneling Through Potential Barrier Distributions with Simple and Fractal Geometries

J.A. Arriaga Hernández, B. Cuevas Otahola, C.M. López López, J. Oliveros Oliveros and O. Valenzuela**
Facultad de Ciencias Físico Matemáticas, Benemérita Universidad Autónoma de Puebla, Puebla, 72000, México.
**Instituto de Astronomía Universidad Nacional Autónoma de México, C.U., Coyoacán, 04510, CDMX.*
jesus.arriagahdz@correo.buap.mx; bolivia.cuevasotahola@viep.com.mx

This work presents an analysis of tunneling across diverse configurations of potential barriers by solving the time-independent Schrödinger equation. We investigated arrays of barriers and wells with controllable spacing and geometry, focusing on tractable distributions and extending the study to multiple architectures, including single and double barriers or mixed arrays encompassing periodic, quasiperiodic, and self-similar (fractal) arrangements. We calculate the energies of states along with the reflection and transmission coefficients to characterize tunneling across the respective distributions. Our results show robust tunneling behavior in these families and reveal fractal features in the energy-resolved transmission and reflection spectra, such as self-similar scaling, hierarchical resonances, and fine structure associated with the formation of minibands that persist under systematic variations of the geometric parameters.

I . Mathematical Induction

Mathematical induction is one of the fundamental forms of proof in mathematics taught from the first semesters, but it is not necessarily simple. Mathematical induction requires a countable sequence of processes, considering and verifying the validity of the first S_1, S_2, and S_3, and constructing the inducible hypothesis for the process S_n. It is then demonstrated for the process S_{n+1}, thereby considering the sequence of processes valid for all n in the (natural) numbers, or even, in some cases, considering zero [1].

II. Potential Barriers

In quantum mechanics, many models of electromagnetic, chemical, and thermodynamic potentials have applications in various areas where potential distributions in **n** dimensions (**nD**) can be related to macroscopic phenomena. For example, in the solid state, these can be electrolytic potentials that maintain the atomic lattice of crystals (generally solids) [2, 3]. The distribution of potential barriers is freely chosen depending on the phenomenon [3, 4], and in our case, we will use simple geometries for the arrangements such that said arrangements admit an inducible distribution for symmetric potentials, where these potentials are the potential barriers [3, 4].

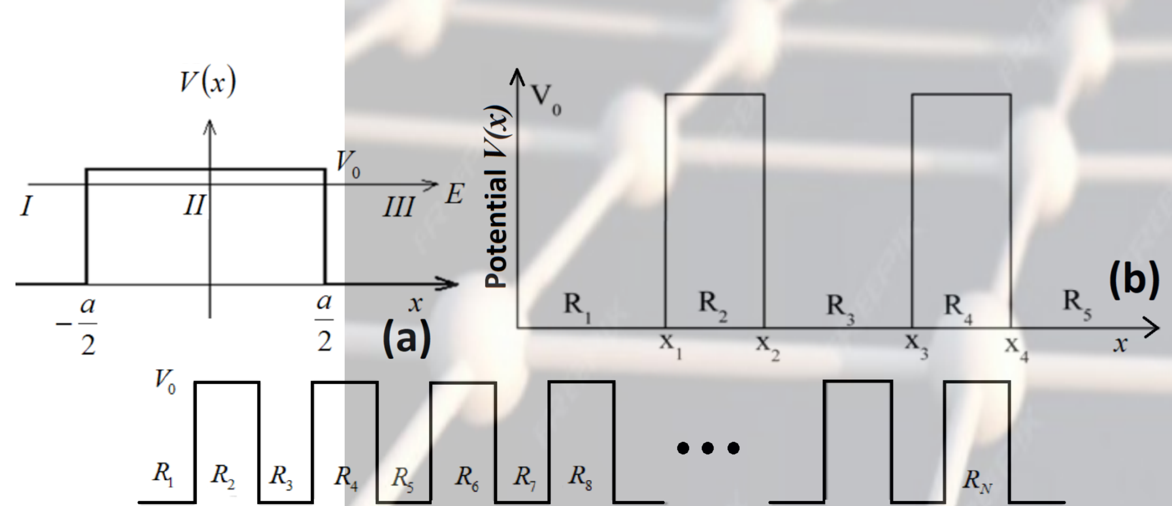


Figure 1: Symmetrical distributions of potential barriers

The different configurations of potential barriers are modeled so that the distributions are integrated into the time-independent Schrödinger equation.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x)+V(x)\psi(x)=E\psi(x)\ , \qquad \text{Ec. (1)}$$

Whose solution is made considering is matrix considering step matrices on the regions R_i and interface matrices entering or leaving the barrier at points x_i.

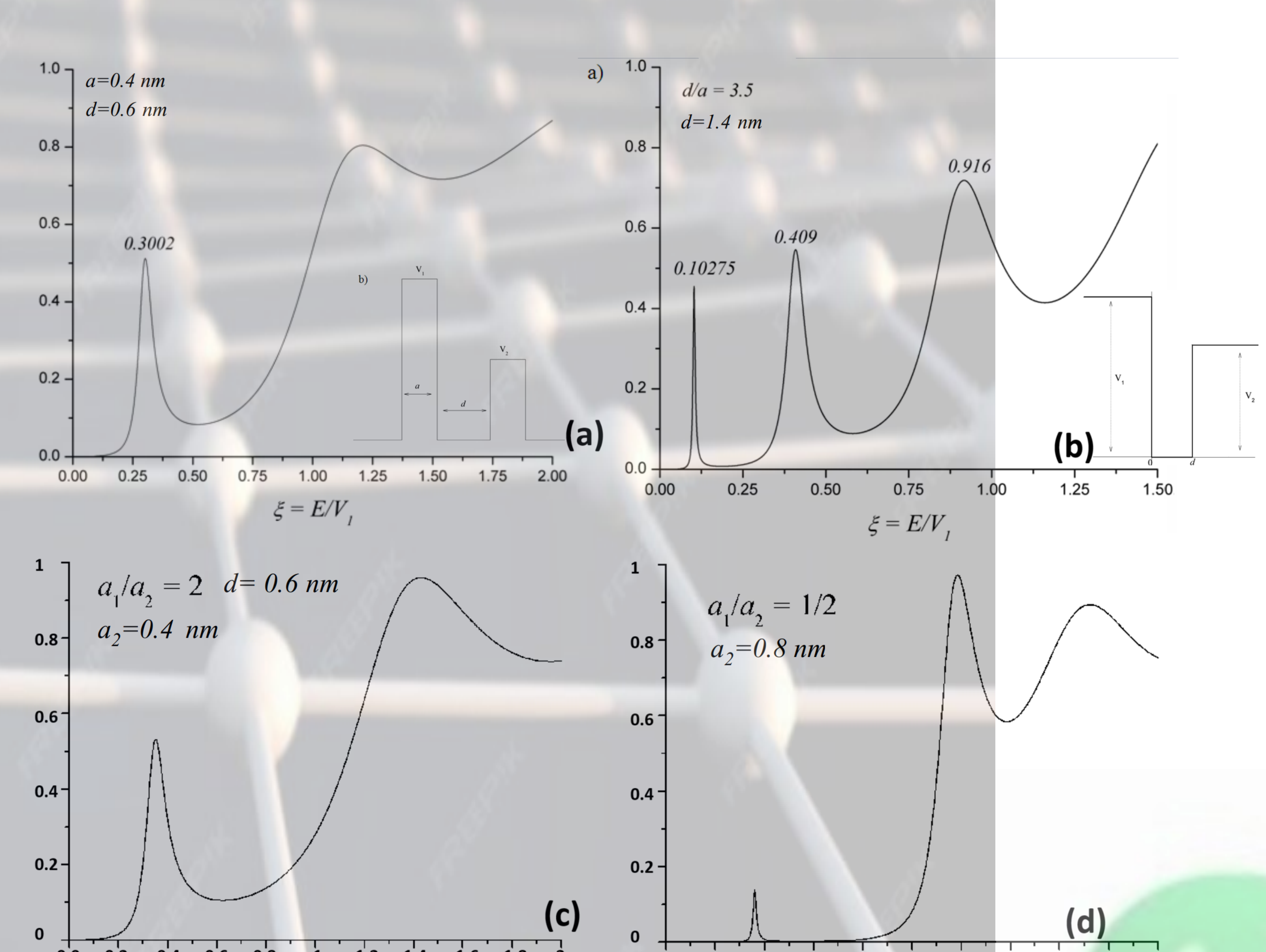
$$\sigma_x = M\sigma_x M\ ,\quad M_1 \equiv I_{23}P_2 I_{12} P_1 \qquad M \equiv M_N M_{N-1}\cdots M_2 M_1$$

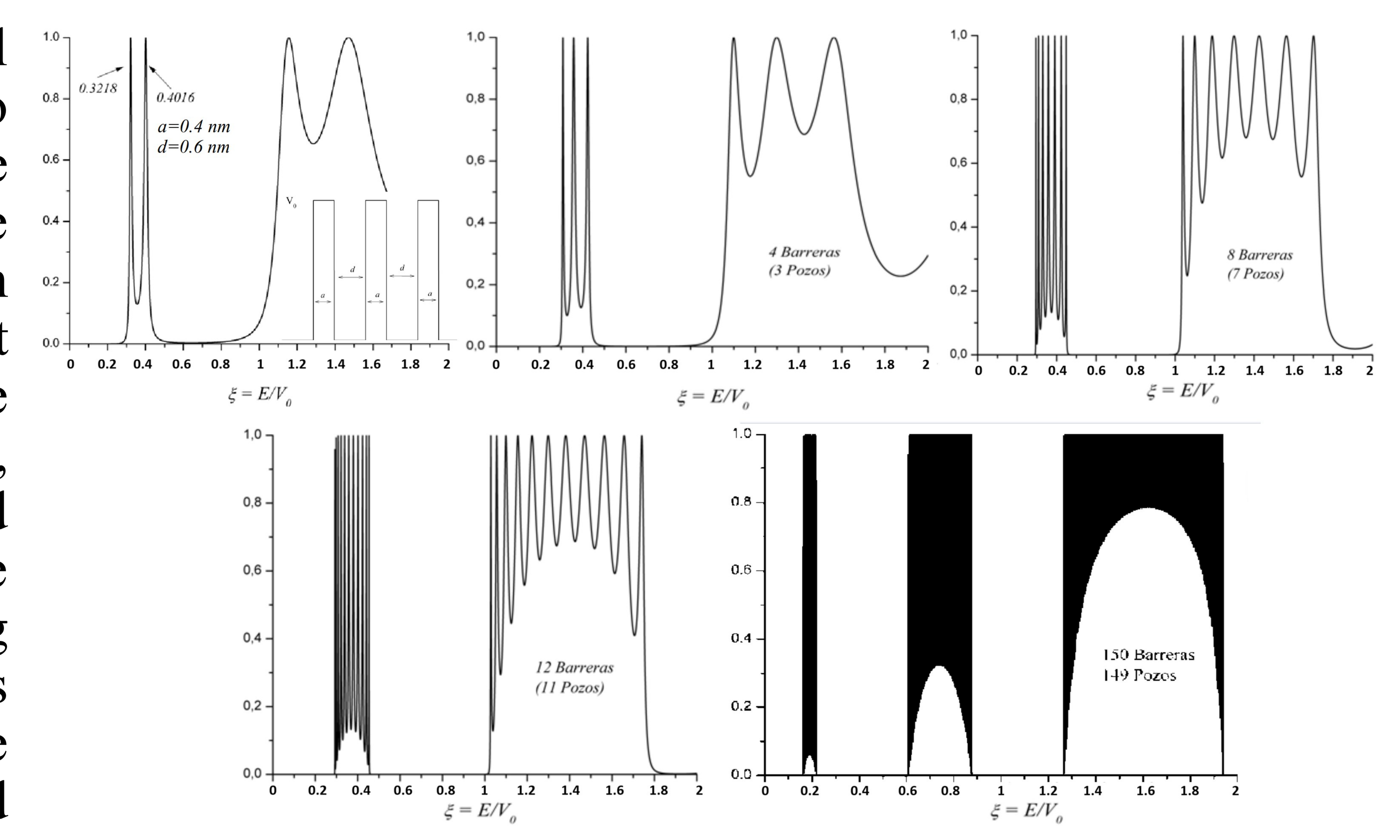
$$T=\frac{1}{|M_{22}|^2} \qquad \text{Ec. (2)}$$

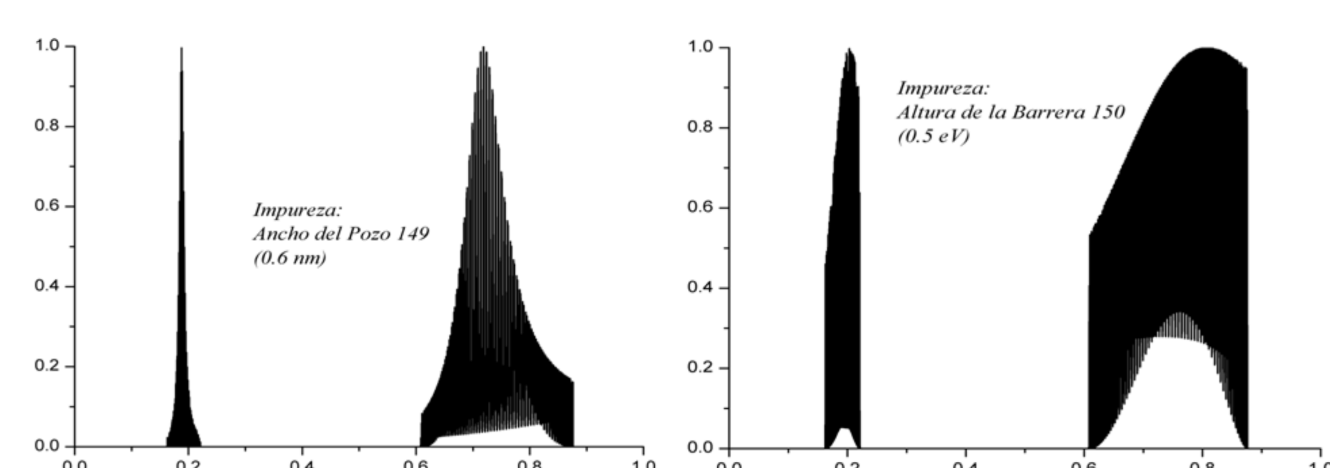
To proof that the different distributions of potential barriers are inducible, which we divide into symmetric and non-symmetric. Within the symmetric ones we focus on potentials of the same magnitude (V_0), same width and distance between the barriers, since each matrix $M_i = I_{i-1}P_{i-1}I_iP_i$ (first interface of the space at the beginning of the barrier I_{i-1} propagation within the barrier, Pi-1, second interface when leaving the barrier, Ii, and the propagation when leaving the barrier, Pi) we satisfactorily demonstrate that the following distributions are inducible: Circular distributions equally spaced according to the diameter of the barrier, square distributions equally spaced according to the width of the barrier and different distances between barriers but constant. In other configurations it was proven that they were not inducible but did show recursive and periodic results.

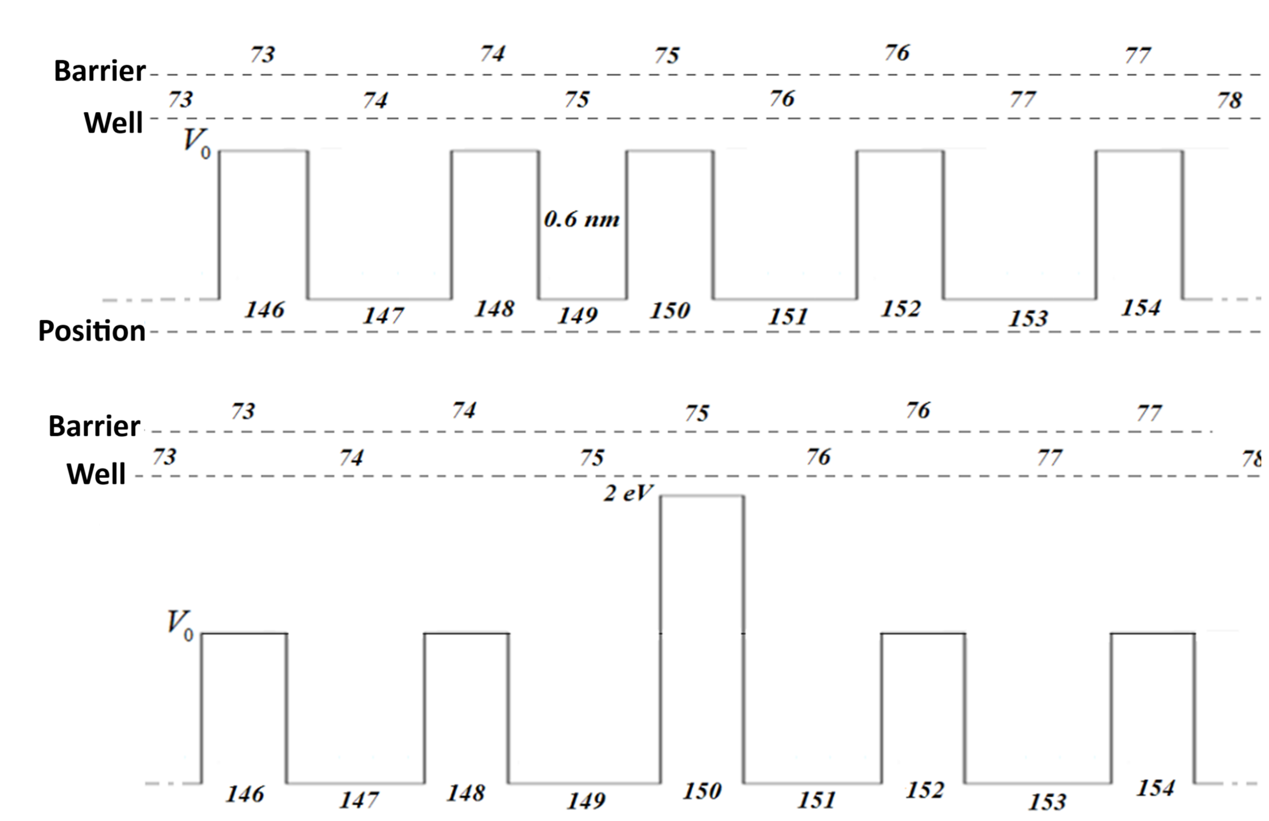
As we demonstrated the mathematical induction, based on the final matrix $M = M_nM_{n-1}\ldots M_2M_1$, then according to Eq. 2 the transmission coefficient was always verified to corroborate the tunnel effect in said arrangements.

III. Results









Conclusions

Inducible sets exhibit certain geometric properties in the results that manifest themselves in experimental events that we are still investigating. In theory, periodic and stable behaviors are observed that can be reproduced. However, in the case of minimal alterations such as doping, the properties are lost, although quantum tunneling occurs.

References

[1].- Henkin, Leon. "On Mathematical Induction." The American Mathematical Monthly 67, no. 4 (1960): 323–38. https://doi.org/10.2307/2308975.

[2].- A. Kole, K. Datta and I. Sengupta, "A New Heuristic for N -Dimensional Nearest Neighbor Realization of a Quantum Circuit," in IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 37, no. 1, pp. 182-192, Jan. 2018, doi: 10.1109/ TCAD.2017.2693284.

[3].– Luis de la Peña, Introducción a la mecánica cuántica 3ª ed. México, FCE, Ediciones Cientificas Universitarias UNAM, 2006.

[4].- Arriaga-Hernández, J. A. (2012). Efecto túnel resonante a través de arreglos de barreras de potencial. [Tesis de licenciatura, Instituto Politécnico Nacional, Escuela Superior de Física y Matemáticas], México. http://repositoriodigital.ipn.mx/ handle/123456789/6036.