Theoretical and phenomenological aspects of the electroweak gauge bosons' Kaluza-Klein modes

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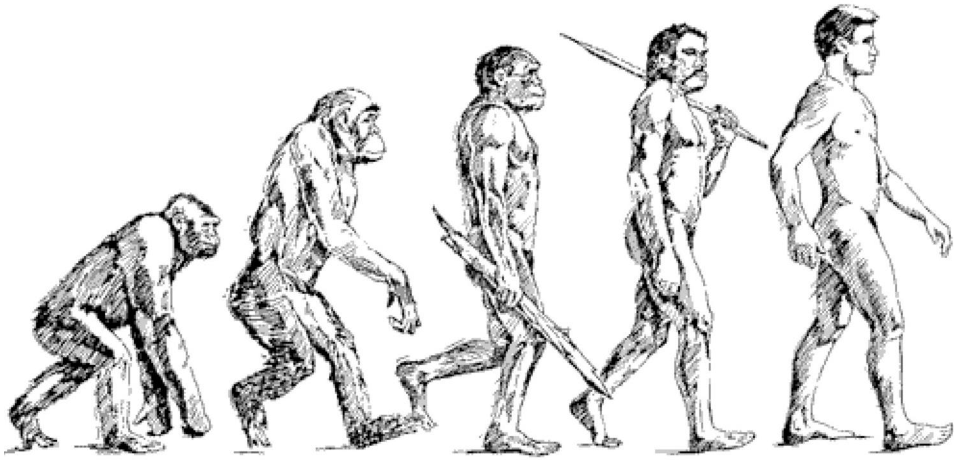
Introduction

Great evolution in our knowledge:



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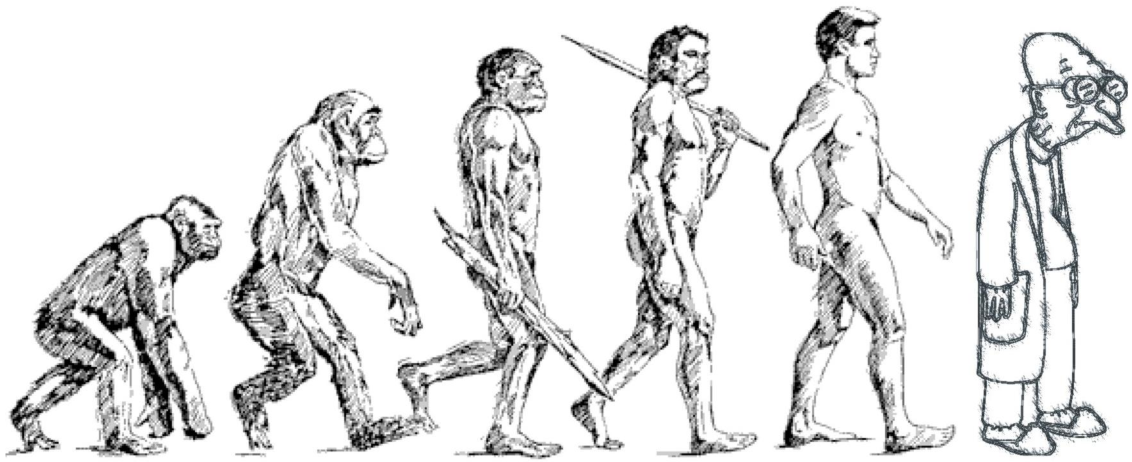
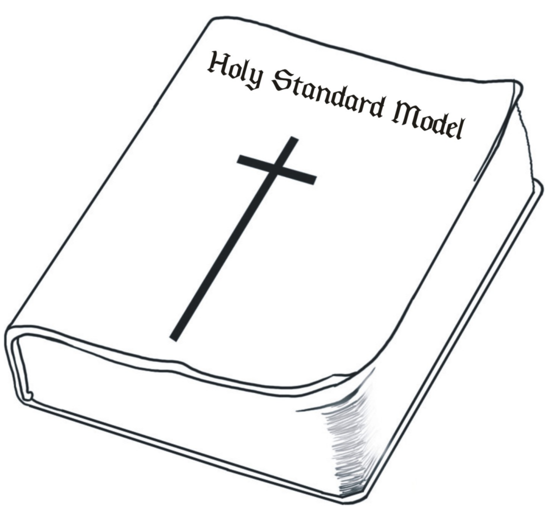


In high energy physics...



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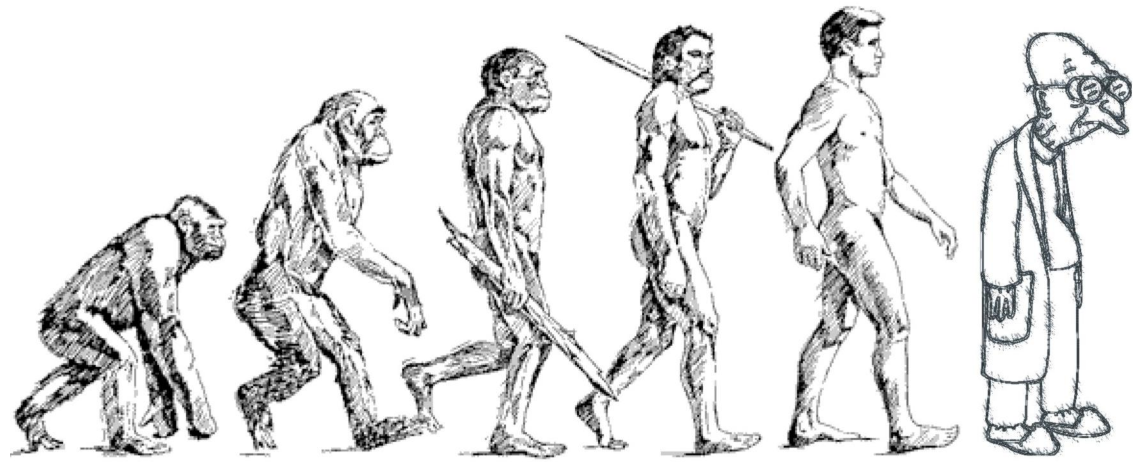
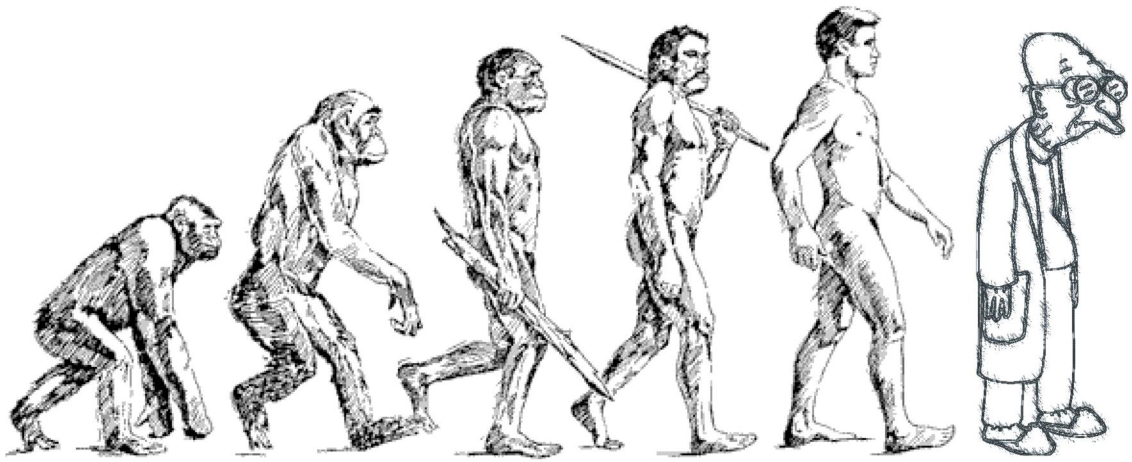


In high energy physics...

But according to nature...

Great evolution in our knowledge:



In high energy physics...

But according to nature...

Large Hadron Collider

Most powerful microscope ever built

Test existing models

Physics never seen before

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- Test existing models
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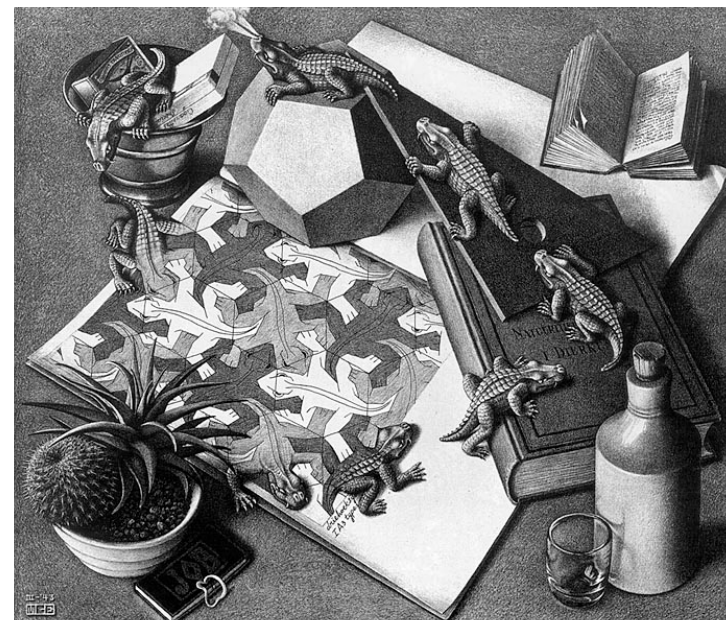


One spatial extra dimension

TeV-sized

Phenomenologically crucial

A five dimensional Maxwell theory

5D QED Lagrangian:

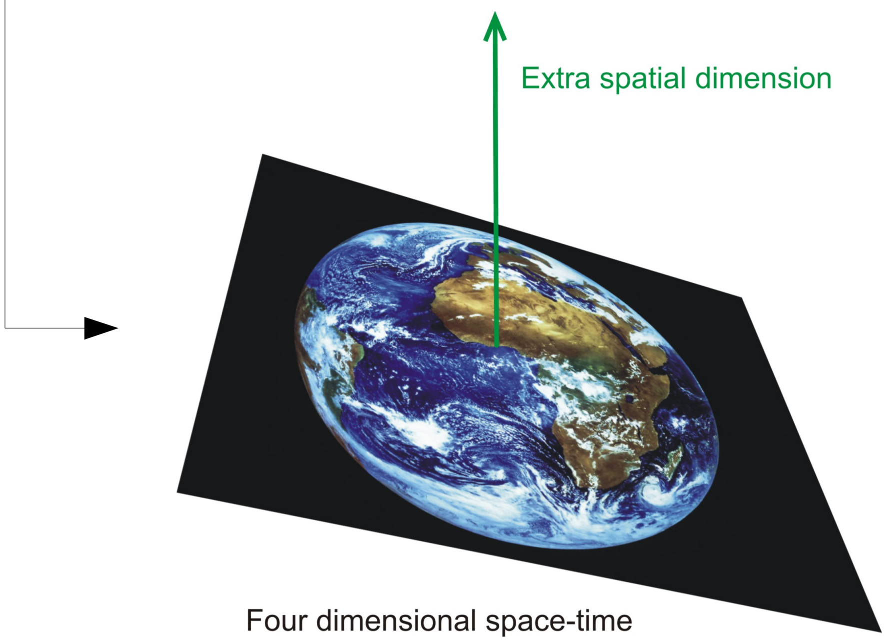
$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\,\mathcal{F}^{MN}(x,y)$$

Symmetry groups

$SO(4,1)$ → 4 spatial + 1 time

$U_5(1)$ → 5 gauge fields

$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\mathcal{F}^{MN}(x,y)$$



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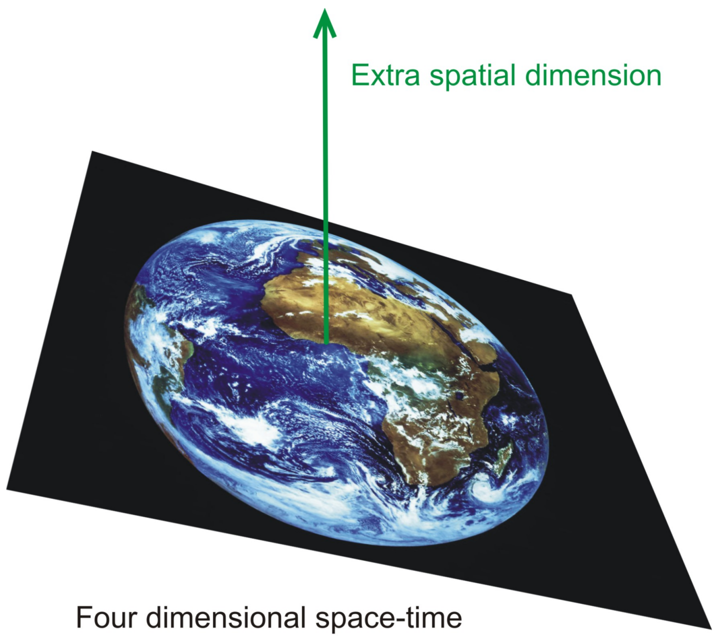
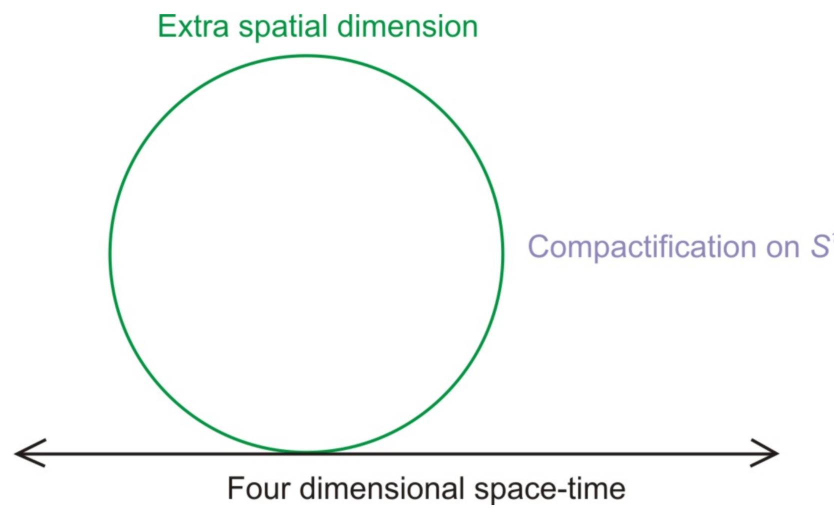
Compactification

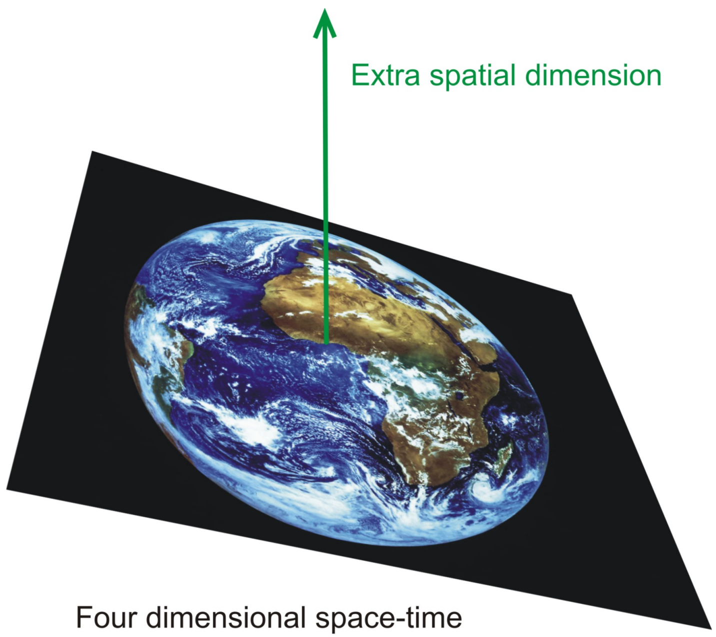


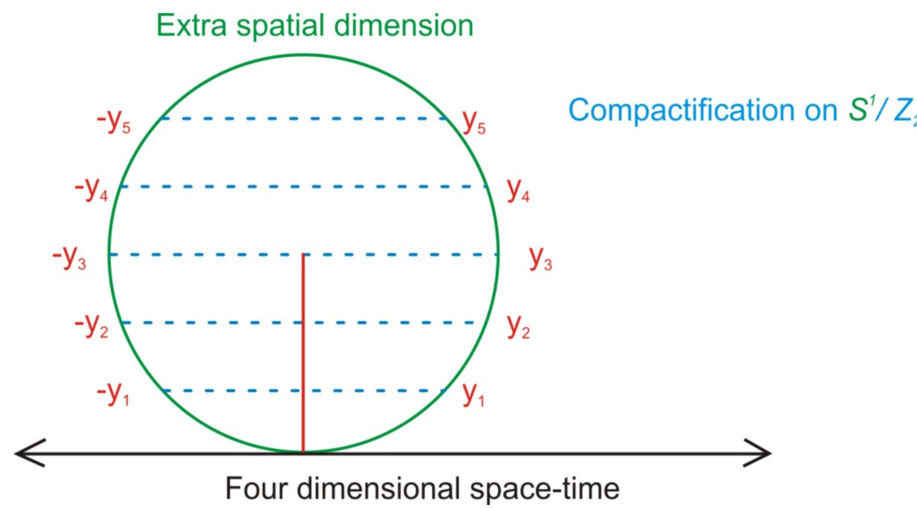


$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\mathcal{F}^{MN}(x,y)$$

Compactification

$$S^1/Z_2$$





Breaking of the symmetry groups

$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\mathcal{F}^{MN}(x,y)$$

Compactification

$$S^1/Z_2$$

Gauge fields

Periodicity:

$$A_M(x,y) = A_M(x,y+2\pi R)$$

Fourier expansions

KK Towers

$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\mathcal{F}^{MN}(x,y)$$

Compactification

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Periodicity:

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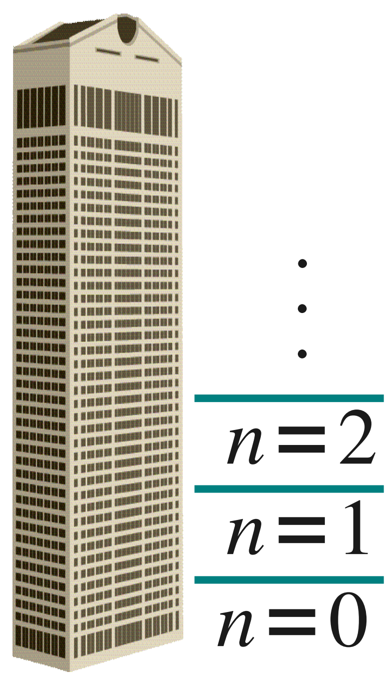

KK modes

$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\mathcal{F}^{MN}(x,y)$$

Compactification

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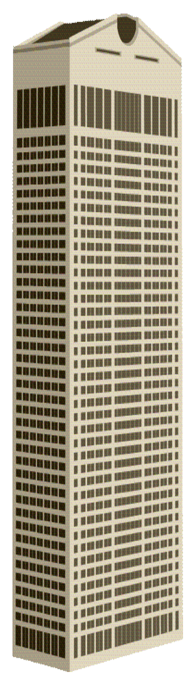

$\vdots$

$n=2$

$n=1$

$n=0$

KK modes

Infinite particles!

$$\mathcal{L}_{5e} = -\frac{1}{4}\mathcal{F}_{MN}(x,y)\mathcal{F}^{MN}(x,y)$$

Compactification

$$S^1/Z_2$$

Gauge fields

Periodicity

Parity:

$$\begin{cases} A_\mu(x,y) = A_\mu(x,-y) \\ A_5(x,y) = -A_5(x,-y) \end{cases}$$

Fourier expansions

Sines or cosines

$$S = \int d^4x \int dy\, \mathcal{L}_{5e}$$

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Integration

$$\mathcal{L}_e = -\frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \sum_{n=1}^{\infty} \left[-\frac{1}{4} F^{(n)}_{\mu\nu} F^{(n)\mu\nu} + \frac{1}{2} \left(\partial_\mu A_5^{(n)} + \frac{n}{R} A_\mu^{(n)} \right) \left(\partial^\mu A_5^{(n)} + \frac{n}{R} A^{(n)\mu} \right) \right]$$

$$S = \int d^4x \int dy\, \mathcal{L}_{5e}$$

Integration

$$\mathcal{L}_e = -\frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \sum_{n=1}^{\infty} \left[-\frac{1}{4} F^{(n)}_{\mu\nu} F^{(n)\mu\nu} + \frac{1}{2} \left(\partial_\mu A_5^{(n)} + \frac{n}{R} A_\mu^{(n)} \right) \left(\partial^\mu A_5^{(n)} + \frac{n}{R} A^{(n)\mu} \right) \right]$$

Ordinary QED

$$S = \int d^4 x \int dy\, \mathcal{L}_{5e}$$

Integration

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Ordinary QED

QED-like terms

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Ordinary QED

QED-like terms

Scalar field

$$S = \int d^4x \boxed{\int dy\, \mathcal{L}_{5e}}$$

Integration

$$\mathcal{L}_e = -\frac{1}{4} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} + \sum_{n=1}^{\infty} \left[-\frac{1}{4} F^{(n)}_{\mu\nu} F^{(n)\mu\nu} + \frac{1}{2} \left(\partial_\mu A_5^{(n)} + \frac{n}{R} A_\mu^{(n)} \right) \left(\partial^\mu A_5^{(n)} + \frac{n}{R} A^{(n)\mu} \right) \right]$$

Ordinary QED

QED-like terms

Scalar field

Mass term

$$S=\int d^4x \int dy\, \mathcal{L}_{5e}$$

Integration

$$\mathcal{L}_e = -\frac{1}{4}F^{(0)}_{\mu\nu}F^{(0)\mu\nu} + \sum_{n=1}^{\infty}\left[-\frac{1}{4}F^{(n)}_{\mu\nu}F^{(n)\mu\nu} + \frac{1}{2}\left(\partial_\mu A_5^{(n)} + \frac{n}{R}A_\mu^{(n)}\right)\left(\partial^\mu A_5^{(n)} + \frac{n}{R}A^{(n)\mu}\right)\right]$$

Ordinary QED

QED-like terms

Scalar field

Mass term

Similar to QED with spontaneous symmetry breaking

$A_\mu^{(n)}$ → Massive particle

$A_5^{(n)}$ → Pseudo-Goldstone boson

And now the Dirac analysis...

Lagrange ⟶ Hamilton

And now the Dirac analysis...

Lagrange → Hamilton

The momenta:

$$\pi^{M}_{(m)} \equiv \frac{\partial \mathcal{L}_e}{\partial\, \partial_0 A^{(m)}_M}$$

Undetermined velocities

Primary constraints, $\phi^{1}_{(m)} \approx 0$

Consistency conditions:

Secondary constraints, $\phi^{2}_{(m)} \approx 0$ → **No more constraints**

Primary constraints:

$$\phi^1_{(0)} = \pi^0_{(0)} \approx 0$$

$$\phi^1_{(m)} = \pi^0_{(m)} \approx 0$$

Secondary constraints:

$$\phi^2_{(0)} = \partial_j \pi^j_{(0)} \approx 0$$

$$\phi^2_{(m)} = \partial_j \pi^j_{(m)} + \frac{m}{R} \pi^5_{(m)} \approx 0$$

All are first class!

Closure:

$$\int d^3\vec{y} \{\phi^i_n(\vec{x}), \phi^j_{(m)}(\vec{y})\} \approx 0$$

Primary constraints:

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$$\phi^1_{(m)} = \pi^0_{(m)} \approx 0$$

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Counting of degrees of freedom

$$dof = \frac{1}{2}\left[(10\mathrm{k}-2)-2(2\mathrm{k})-0\right] = 3\mathrm{k}-1$$

Primary constraints:

$$\phi^{1}_{(0)} = \pi^{0}_{(0)} \approx 0$$

$$\phi^{1}_{(m)} = \pi^{0}_{(m)} \approx 0$$

Secondary constraints:

$$\phi^{2}_{(0)} = \partial_j \pi^{j}_{(0)} \approx 0$$

$$\phi^{2}_{(m)} = \partial_j \pi^{j}_{(m)} + \frac{m}{R} \pi^{5}_{(m)} \approx 0$$

Counting of degrees of freedom

Dynamical variables

First class constraints

Second class constraints

$$dof = \frac{1}{2}\left[(10k - 2) - 2(2k) - 0 \right] = 3k - 1$$

Primary constraints:

$$\phi^{1}_{(0)} = \pi^{0}_{(0)} \approx 0$$

$$\phi^{1}_{(m)} = \pi^{0}_{(m)} \approx 0$$

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Counting of degrees of freedom

Dynamical variables — First class constraints — Second class constraints

$$dof = \frac{1}{2}\left[\left(10\mathrm{k}-2\right) - 2\left(2\mathrm{k}\right) - 0\right] = 3\mathrm{k} - 1$$

$\Rightarrow$

- Zero mode → Two degrees of freedom
- Each KK mode → Three degrees of freedom

Gauge generator:

$$\mathcal{G} = \int d^3\vec{x}\left[(\partial_0 \epsilon^{(0)})\phi^1_{(0)} - \epsilon^{(0)}\phi^2_{(0)} + \sum_{n=1}^{\infty}\left((\partial_0 \epsilon^{(n)})\phi^1_{(n)} - \epsilon^{(n)}\phi^2_{(n)}\right)\right]$$

Gauge transformations:

Gauge generator:

$$\mathcal{G} = \int d^3\vec{x}\left[(\partial_0 \epsilon^{(0)})\phi^1_{(0)} - \epsilon^{(0)}\phi^2_{(0)} + \sum_{n=1}^{\infty}\left((\partial_0\epsilon^{(n)})\phi^1_{(n)} - \epsilon^{(n)}\phi^2_{(n)}\right)\right]$$

Gauge transformations:

$$A_\mu^{(m)} \to A_\mu^{(m)} + \partial_\mu \epsilon^{(m)}$$

$$A_5^{(m)} \to A_5^{(m)} - \frac{m}{R}\epsilon^{(m)}$$

All of the fields are gauge fields!

$$\mathcal{L} = \mathcal{L}_e - \frac{1}{2}\xi\left(\partial_\mu A^{(0)\mu}\right)^2$$

Gauge fixing for KK modes

$$A_\mu^{(n)} \rightarrow A_\mu^{(n)} - \frac{R}{n}\partial_\mu A_5^{(n)}$$

$$\mathcal{L}_e = -\frac{1}{4}F_{\mu\nu}^{(0)}F^{(0)\mu\nu} - \frac{1}{2\xi}\left(\partial_\nu A^\mu\right)^2 + \sum_{n=1}^{\infty}\left[-\frac{1}{4}F_{\mu\nu}^{(n)}F^{(n)\mu\nu} + \frac{1}{2}\frac{n^2}{R^2}A_\mu^{(n)}A^{(n)\mu}\right]$$

No more $A_5^{(n)}$ → $A_5^{(n)}$ is a gauge field...

$$\mathcal{L} = \mathcal{L}_e - \frac{1}{2}\xi(\partial_\mu A^{(0)\mu})^2$$

Gauge fixing for KK modes

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No more $A_5^{(n)}$ → $A_5^{(n)}$ is a gauge field...

but a non-physical one!

A five dimensional Yang-Mills Theory

The Yang-Mills 5D Lagrangian:

$$\mathcal{L}_{YM5} = -\frac{1}{4}\mathcal{F}_{MN} \cdot \mathcal{F}^{MN}$$

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$$\mathcal{L}_{YM5} = -\frac{1}{4}\mathcal{F}_{MN}\cdot\mathcal{F}^{MN}$$

$$\mathcal{F}^{a}_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 (A_M \times A_N)^a$$

5D coupling constant

The Yang-Mills 5D Lagrangian:

$$\mathcal{L}_{YM5} = -\frac{1}{4}\mathcal{F}_{MN}\cdot\mathcal{F}^{MN}$$

Compactification

$$\begin{cases} A_M^a(x,y) = A_M^a(x,y+2\pi R) \\ A_\mu^a(x,y) = A_\mu^a(x,-y) \\ A_5^a(x,y) = -A_5^a(x,-y) \end{cases}$$

Fourier expansions

The Yang-Mills 5D Lagrangian:

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Integration

Fourier expansions

Complicated effective Lagrangian

The Yang-Mills 5D Lagrangian:

$$\mathcal{L}_{YM5} = -\frac{1}{4}\mathcal{F}_{MN}\cdot\mathcal{F}^{MN}$$

Compactification

$$A^a_M(x,y) = A^a_M(x,y+2\pi R)$$
$$A^a_\mu(x,y) = A^a_\mu(x,-y)$$
$$A^a_5(x,y) = -A^a_5(x,-y)$$

Integration

Fourier expansions

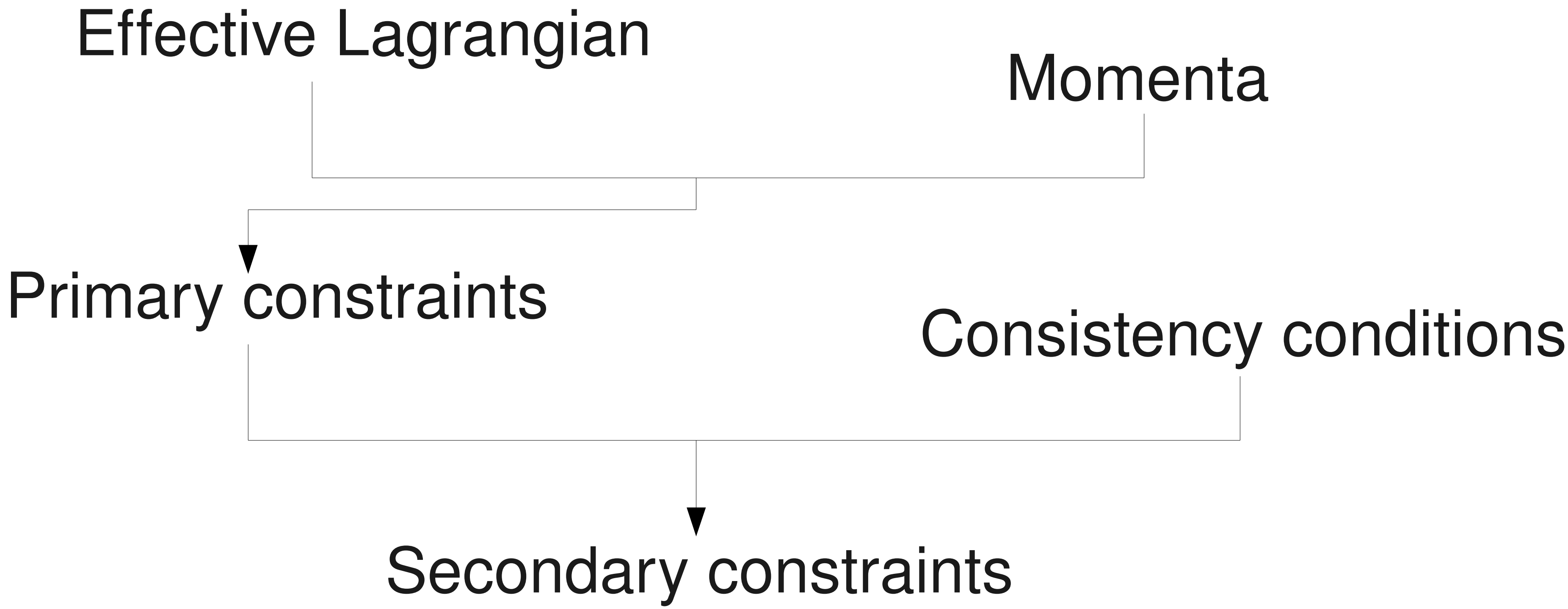
Complicated effective Lagrangian

- Covariant derivative
- Interactions involving different KK modes
- Interactions involving three and four fields

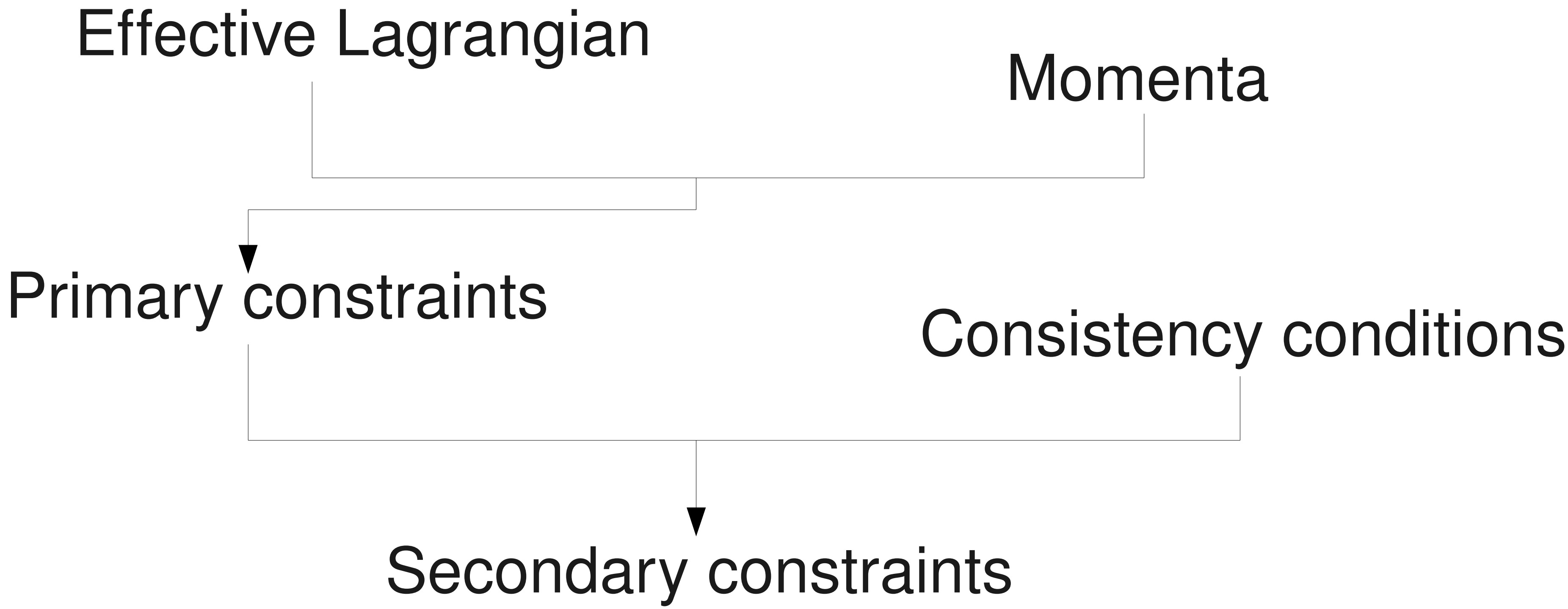
Effective Lagrangian

Momenta

Primary constraints


Effective Lagrangian
Momenta
Primary constraints
Consistency conditions
Secondary constraints

And we have...



And we have...

Primary constraints

$$\phi^{1}_{(0)l} = \pi^{0}_{(0)l} \approx 0$$

$$\phi^{1}_{(n)l} = \pi^{0}_{(n)l} \approx 0$$

Secondary constraints

$$\phi^{2}_{(0)l} = \left(\mathcal{D}_j \pi^{j}_{(0)} + g \sum_{n=1}^{\infty} \left[A^{(n)}_{j} \times \pi^{j}_{(n)} + A^{(n)}_{5} \times \pi^{5}_{(n)} \right] \right)^{l}$$

$$\phi^{2}_{(n)l} = very\ long\ expression$$

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$$\phi^{2}_{(n)l} = very\ long\ expression$$

Closure fulfilled...

Primary constraints

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Secondary constraints

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$$\phi^{2}_{(n)l} = very\ long\ expression$$

Closure fulfilled...

but $\{\phi^{1}_{(0)l}, \phi^{1}_{(n)l}, \phi^{2}_{(0)l}, \phi^{2}_{(n)l}\}$ does not seem to define a closed algebra

- Mass endowing effects by means of symmetry breaking
- Self-interactions

To be continued...