

HIGH PRECISION AT HADRON MACHINES

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OUTLINE

- Introduction to high accuracy at the LHC
- High precision at fixed order
 - Perturbative calculations
 - Non-perturbative determinations
- Conclusions

-Physics at the LHC -

- LHC experiments are delivering more and more data to the HEP community.
- More data sets improve the accuracy of all observables.
- And, when the physics is hidden in small effects, accuracy is crucial to claim discovery.
- The new 750 GeV resonance !

-High Precision -

- LO do not even provides the order of magnitude of the cross section.
- Virtual corrections could have an enormous impact for the LHC physics program.
- High precision requires the computation of new Feynman diagrams.



Higgs discovery

- In 2012 at CERN, ATLAS and CMS announced a 5sigma evidence of a resonance around 125 GeV.
- The claim was that the bosonic particle is the SM-Higgs.
- Determination of properties of new particles and New Physics requires precision measurements.



-Hadron machines —

- Hadron machines are dominated by gluon densities (QCD).
- Fundamental particles are only detected indirectly.



- New Physics has to be disentangled from Standard Model physics. High precision in SM calculations.
- Hadron machines are dominated by QCD effects. Exact solutions of QCD are not known, then pQCD helps to compute scattering amplitudes.
- However, high accuracy on LHC observables requires higher orders in pQCD.
- Besides, high level of accuracy is reached when the perturbative and non-perturbative pieces are under control at fixed order in pQCD.

-Setup at hadron machines -

• How to translate theoretical predictions to experimental measurements ?



-Theory meets experiment -

• Precision measurements use accurate theoretical predictions.



 Experimental results needs Monte Carlo simulations in order to compare with nature.

-Theory meets experiment -

• Precision measurements use accurate theoretical predictions.



 Experimental results needs Monte Carlo simulations in order to compare with nature.

and for the next run(s)

Moreover, future observables need accurate Monte Carlo simulations.



 Relevant contributions will only be explained when NNLO Monte Carlo simulations are implemented.

PERTURBATIVE TOOLS

-Dimensional regularisation (DREG) -

- DREG promotes 4-dimensional integrals to d-dimensional integrals $(d = 4 2\epsilon)$.
- Divergences appear like poles in the dimensional parameter ϵ .
- Loop integrals generate UV and IR poles.
- UV (High energy region) poles are cured with proper counterterms.
- IR (Low energy region) poles are cured by adding real emission processes.

Camplitude level



• The total cross section is computed as,

$$\sigma = \sigma^{(LO)} + \alpha_S(\mu)\sigma^{(NLO)} + \alpha_S^2(\mu)\sigma^{(NNLO)} + \cdots$$

where

$$\sigma^{(NLO)} = \int_{\Omega} d\sigma^{V} + \int_{\Omega+1} d\sigma^{R}$$
$$\sigma^{(NNLO)} = \int_{\Omega} d\sigma^{VV} + \int_{\Omega+1} d\sigma^{RV} + \int_{\Omega+2} d\sigma^{RR}$$

• and divergences are everywhere.

-Theoretical issues

- Integrands are usually lengthy.
- The number of Feynman diagrams increase enormously when high accuracy is required.
- Monte Carlo simulations first compute the poles and shows the exact cancellation at integral-level. Then, it computes the number of integrals separately (2 at NLO, 3 at NNLO, etc.).
- New Physics searches have to be done at the highest posible accuracy.
- New methods for higher order calculations are extremely important.

-Loop-Tree duality —

Massive one-loop scalar integrals are,



 where the +i0 prescription establishes that particles are going forward in time. The solution of the integrals are known by the Cauchy residues theorem.





However, by using advanced propagators,





• LTD at one loop establishes then

$$L^{(1)}(p_1, \cdots, p_N) = -\sum_{\ell_1} \tilde{\delta}(q_i) \prod_{\substack{j=1\\ j \neq i}}^N G_D(q_i; q_j)$$

where Feynman propagators are transformed to dual propagators.

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot (q_j - q_i)}$$

- $\tilde{\delta}(q_i) = 2\pi i \theta(q_{i,0}) \delta(q_i^2 m_i^2)$ and sets internal lines on-shell and in the positive energy mode.
- LTD modify the +i0 prescription, instead of having multiple cuts like in the Feynman Tree Theorem.
- η^{μ} is a future-like vector, for simplicity we take $\eta^{\mu} = (1, \mathbf{0})$. In fact, the only relevance is the sign in the prescription.

-Numerical implementation

- Faster computations are needed for the Montecarlo simulations for the LHC observables.
- Using LTD the standard methods become time consuming
- (S. Buchta, et al. , arXiv: 1510.00187)

	Rank	Tensor Pentagon	Real Part	Imaginary Part	Time [s]
P16	2	LoopTools	$-1.86472 imes 10^{-8}$		
		SecDec	$-1.86471(2) imes 10^{-8}$		45
		LTD	$-1.86462(26) imes 10^{-8}$		1
P17	3	LoopTools	$1.74828 imes 10^{-3}$		
		SecDec	$1.74828(17) imes 10^{-3}$		550
		LTD	$1.74808(283) imes 10^{-3}$		1
P18	2	LoopTools	$-1.68298 imes 10^{-6}$	$+i \; 1.98303 imes 10^{-6}$	
		SecDec	$-1.68307(56) imes 10^{-6}$	$+i \ 1.98279(90) imes 10^{-6}$	66
		LTD	$-1.68298(74) imes 10^{-6}$	$+i \ 1.98299(74) imes 10^{-6}$	36
P19	3	LoopTools	$-8.34718 imes 10^{-2}$	$+i \; 1.10217 imes 10^{-2}$	
		SecDec	$-8.33284(829) imes 10^{-2}$	$+i \ 1.10232(107) imes 10^{-2}$	1501
		LTD	$-8.34829(757) imes 10^{-2}$	$+i \ 1.10119(757) imes 10^{-2}$	38

	Rank	Tensor Hexagon	Real Part	Imaginary Part	Time[s]
P20	1	SecDec	$-1.21585(12) \times 10^{-15}$		36
		LTD	$-1.21552(354) \times 10^{-15}$		6
P21	3	SecDec	$4.46117(37) \times 10^{-9}$		5498
		LTD	$4.461369(3) imes 10^{-9}$		11
P22	1	SecDec	$1.01359(23) \times 10^{-15}$	$+i \ 2.68657(26) \times 10^{-15}$	33
		LTD	$1.01345(130) imes 10^{-15}$	$+i \ 2.68633(130) \times 10^{-15}$	72
P23	2	SecDec	$2.45315(24) imes 10^{-12}$	$-i \ 2.06087(20) \times 10^{-12}$	337
		LTD	$2.45273(727) imes 10^{-12}$	$-i \ 2.06202(727) \times 10^{-12}$	75
P24	3	SecDec	$-2.07531(19) \times 10^{-6}$	$+i 6.97158(56) \times 10^{-7}$	14280
		LTD	$-2.07526(8) imes 10^{-6}$	$+i \ 6.97192(8) \times 10^{-7}$	85

- This results shows have been implemented for several data points for tensor pentagons and hexagons.
- Integrals considering massive internal lines were computed numerically.
- The results using LTD are, in some cases, four order of magnitudes faster than SecDec.
- What about in a physical process ?

$\gamma^* \to q\bar{q} \ at \ NLO \ in \ QCD$

• In this well known process, the Feynman diagrams are



- where the process add more structure to the integrals. In general, virtual and real corrections have numerators.
- In this case, for the virtual correction is given by,

$$\langle \mathcal{M}_{q\bar{q}}^{(0)} | \mathcal{M}_{q\bar{q}}^{(1)} \rangle = g_{\rm S}^2 C_F | \mathcal{M}_{q\bar{q}}^{(0)} |^2 \frac{4}{s_{12}} \int_{\ell} \left(\prod_{i=1}^3 G_F(q_i) \right)$$

$$\times \epsilon \left[(2+\epsilon)(q_2 \cdot p_1)(q_3 \cdot p_2) - \epsilon \left((q_2 \cdot p_2)(q_3 \cdot p_1) + \frac{s_{12}}{2}(q_2 \cdot q_3) \right) \right]$$

• and for the real correction is,

$$\sigma_R^{(1)} = \sigma^{(0)} \frac{(4\pi)^{\epsilon-2}}{\Gamma(1-\epsilon)} g_{\rm S}^2 C_F \left(\frac{s_{12}}{\mu^2}\right)^{-\epsilon} \int_0^1 dy'_{1r} \int_0^{1-y'_{1r}} dy'_{2r} (y'_{1r} y'_{2r} y'_{12})^{-\epsilon}$$

$$\times \left[4 \left(\frac{y_{12}'}{y_{1r}' y_{2r}'} - \epsilon \right) + 2(1 - \epsilon) \left(\frac{y_{2r}'}{y_{1r}'} + \frac{y_{1r}'}{y_{2r}'} \right) \right]$$

• Using DREG, the result is,

$$\sigma_{V}^{(1)} = \sigma^{(0)} c_{\Gamma} g_{\rm S}^{2} C_{F} \left(\frac{s_{12}}{\mu^{2}}\right)^{-\epsilon} \left[-\frac{4}{\epsilon^{2}} - \frac{6}{\epsilon} - 16 + 2\pi^{2} + \mathcal{O}(\epsilon)\right]$$
$$\sigma_{R}^{(1)} = \sigma^{(0)} c_{\Gamma} g_{\rm S}^{2} C_{F} \left(\frac{s_{12}}{\mu^{2}}\right)^{-\epsilon} \left[\frac{4}{\epsilon^{2}} + \frac{6}{\epsilon} + 19 - 2\pi^{2} + \mathcal{O}(\epsilon)\right]$$

• Then,

$$\sigma = \sigma^{(0)} \left(1 + 3C_F \frac{\alpha_{\rm S}}{4\pi} + \mathcal{O}(\alpha_{\rm S}^2) \right)$$

Remarks:

- IR behaviour is quite similar to the scalar case, therefore the same mapping is performed.
- There is no need of tensor reduction, no need of Gram determinants.
- Two point function of massless particles are usually ignored because is scaleless.
- In fact, this integral is zero because IR and UV poles cancels.
- In the LTD, there is an identification of IR and UV regions, therefore it has to be consider at the integrand level.

• Following the procedure described, it is possible to find 4dimensional representations for the cross sections, resulting:

$$\begin{aligned} \widetilde{\sigma}_{1}^{(1)} &= \sigma^{(0)} \frac{\alpha_{S}}{4\pi} C_{F} \int_{0}^{1} d\xi_{1,0} \int_{0}^{1/2} dv_{1} \, 4 \, \mathcal{R}_{1}(\xi_{1,0}, v_{1}) \left[2 \left(\xi_{1,0} - (1 - v_{1})^{-1} \right) - \frac{\xi_{1,0}(1 - \xi_{1,0})}{(1 - (1 - v_{1}) \, \xi_{1,0})^{2}} \right] \\ \widetilde{\sigma}_{2}^{(1)} &= \sigma^{(0)} \frac{\alpha_{S}}{4\pi} C_{F} \int_{0}^{1} d\xi_{2,0} \int_{0}^{1} dv_{2} \, 2 \, \mathcal{R}_{2}(\xi_{2,0}, v_{2}) \, (1 - v_{2})^{-1} \left[\frac{2 \, v_{2} \, \xi_{2,0} \left(\xi_{2,0}(1 - v_{2}) - 1 \right)}{1 - \xi_{2,0}} \right] \\ &- 1 + v_{2} \, \xi_{2,0} + \frac{1}{1 - v_{2} \, \xi_{2,0}} \left(\frac{(1 - \xi_{2,0})^{2}}{(1 - v_{2} \, \xi_{2,0})^{2}} + \xi_{2,0}^{2} \right) \right], \end{aligned}$$

$$\begin{split} \overline{\sigma}_{\mathrm{V}}^{(1)} &= \sigma^{(0)} \frac{\alpha_S}{4\pi} C_F \int_0^\infty d\xi \int_0^1 dv \left\{ -2 \left(1 - \mathcal{R}_1(\xi, v)\right) v^{-1} (1 - v)^{-1} \frac{\xi^2 (1 - 2v)^2 + 1}{\sqrt{(1 + \xi)^2 - 4v \,\xi}} \right. \\ &+ 2 \left(1 - \mathcal{R}_2(\xi, v)\right) \left(1 - v\right)^{-1} \left[2v \,\xi \left(\xi (1 - v) - 1\right) \left(\frac{1}{1 - \xi + i0} + i\pi \delta (1 - \xi)\right) - 1 + v \,\xi \right] \\ &+ 2 \,v^{-1} \left(\frac{\xi (1 - v) (\xi (1 - 2v) - 1)}{1 + \xi} + 1 \right) - \frac{(1 - 2v) \,\xi^3 \left(12 - 7m_{\mathrm{UV}}^2 - 4\xi^2\right)}{(\xi^2 + m_{\mathrm{UV}}^2)^{5/2}} \\ &- \frac{2 \,\xi^2 (m_{\mathrm{UV}}^2 + 4\xi^2 (1 - 6v(1 - v)))}{(\xi^2 + m_{\mathrm{UV}}^2)^{5/2}} \right\}, \end{split}$$

• Then, this integrals in 4D are solvable analytically, resulting:

$$\tilde{\sigma}_{1}^{(1)} = \sigma^{(0)} \frac{\alpha_{\rm S}}{4\pi} C_F \left(19 - 32 \log(2) \right),$$

$$\tilde{\sigma}_{2}^{(1)} = \sigma^{(0)} \frac{\alpha_{\rm S}}{4\pi} C_F \left(-\frac{11}{2} + 8 \log(2) - \frac{\pi^2}{3} \right),$$

$$\bar{\sigma}_{V}^{(1)} = \sigma^{(0)} \frac{\alpha_{\rm S}}{4\pi} C_F \left(-\frac{21}{2} + 24 \log(2) + \frac{\pi^2}{3} \right).$$

- Thus: $\tilde{\sigma}_1^{(1)} + \tilde{\sigma}_2^{(1)} + \bar{\sigma}_V^{(1)} = \sigma^{(0)} 3 C_F \frac{\alpha_{\rm S}}{4\pi}$
- Computation of multi-legs and NNLO corrections are doable within the LTD.

NON PERTURBATIVE DISTRIBUTIONS

Motivation of FFs

- Input for helicities PDFs and transverse momentum PDFs.
- Necessary for a complete understanding of hadron production in presence of nuclear medium.
- Heavy Ion programs: RHIC and LHC.



-Theory & Uncertainties-

Basic idea of hadronization: Cascade fragmentationRank2I



rank = 1 : "valence", e.g.

$$u \rightarrow \pi^+$$

rank > 2 : "sea", e.g.
 $u \rightarrow \pi^-$

- Theory framework: "independent fragmentation".
- QCD approach based on factorization.
- e+e-: first data used for extracting FFs with LEP data (BKK '95 and KRE '00).

The fitters

Name	Ref.	Species	Error	z _{min}	Q^2 (GeV ²)
AKK	[4]	$\pi^{\pm}, K^{\pm}, K^0_s, p, p \Lambda, \Lambda$	no	0.1	$2-4\cdot 10^4$
AKK08	[5]	$\pi^{\pm}, K^{\pm}, K^0_s, p, p \Lambda, \Lambda$	yes	0.05	$2 - 4 \cdot 10^4$
BKK	[6]	$\pi^+ + \pi^-, \pi^0, K^+ + K^-, K^0 + K^0, h^+ + h^-$	no	0.05	2 - 200
BFG	[7]	γ	no	10^{-3}	$2 - 1.2 \cdot 10^4$
BFGW	[8]	h^{\pm}	yes ¹	10^{-3}	$2 - 1.2 \cdot 10^4$
CGRW	[9]	π^0	no	10^{-3}	$2 - 1.2 \cdot 10^4$
DSS	[10,11]	$\pi^{\pm}, K^{\pm}, p, p, h^{\pm}$	yes ²	0.05-0.1	$1 - 10^5$
DSV	[12]	polarized and unpolarized Λ	no	0.05	$1 - 10^4$
GRV	[13]	γ	no	0.05	≥ 1
HKNS	[14]	$\pi^{\pm}, \pi^{0}, K^{\pm}, K^{0} + K^{0}, n, p + p$	yes	0.01 – 1	$1 - 10^{8}$
KKP	[15]	$\pi^+ + \pi^-, \pi^0, K^+ + K^-, K^0 + K^0, p + p, n + n, h^+ + h^-$	no	0.1	$1 - 10^4$
Kretzer	[16]	$\pi^{\pm}, K^{\pm}, h^+ + h^-$	no	0.01	$0.8 - 10^{6}$
l					

- AKK08: e+e- and pp data / Isospin symmetry for pions.
- HKNS: e+e- data only / Hessian method for uncertainties.



- DSS fit arrived to a data-driven separation of individual parton-to-pion fragmentations.
- Large charge symmetry violation between u- and dquarks FFs (~10%).
- Gluon FFs was constrained for the first time with BNL-RHIC data.
- Lagrange multiplier technique was used for estimating uncertainties.

FFs in data: e+e- SIA

• The distribution is given terms of the structure functions,

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^h}{dz} = \frac{\sigma^0}{\sum_q \hat{e}_q^2} \left[2F_1^h(z,Q^2) + F_L^h(z,Q^2)\right]$$

$$2F_1^h(z,Q^2) = \sum_q \hat{e}_q^2 \left\{ \left[D_q^h + D_{\bar{q}}^h \right](z,Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \left[C_q^1 \otimes \left[D_q^h + D_{\bar{q}}^h \right] + C_g^1 \otimes D_g^h \right](z,Q^2) \right\}$$

- Not possible to separate charge and flavour only with SIA.
- Only have information of the singlet.



FFs in data: SIDIS

Distributions for SIDIS are given by,

 $(\overline{O}$

$$\frac{d\sigma^{h}}{dxdydz^{h}} = \frac{2\pi\alpha_{s}(Q^{2})}{Q^{2}} \left[\frac{1 + (1 - y)^{2}}{y} 2F_{1}^{h} + \frac{2(1 - y)}{y}F_{L}^{h} \right] (x, z_{h}, Q^{2})$$

$$2F_{1}^{h}(x, z_{h}, Q^{2}) = \sum_{q, \bar{q}} \hat{e}_{q}^{2} \cdot q(x, Q^{2})D_{q}^{h}(z_{h}, Q^{2})$$

@NLO, all coefficients are known:

Altarelli et al. '79, Furmanski, Petronzio '82, de Florian, Stratmann, Vogelsang '98

- Charge and flavour separation is achieved by including SIDIS.
- However, gluon FF is not well constrained by SIA and SIDIS data.



FFs in data: Hadron collisions

• The general picture is:



• Therefore, transverse momentum distribution is given by:

$$\frac{d\sigma(pp \to hX)}{dp_T d\eta} = \sum_{i,j,k} \int dx_1 dx_2 dz \left[f_i^P(x_1,\mu_f) f_j^P(x_2,\mu_f) D_k^h(z,\mu_f') \frac{d\hat{\sigma}(ij \to kX')}{dp_T d\eta} \right]$$

- It also allows charge and flavour separation.
- It contains large contributions from gluons.

DSS vs the new fit

- Number of parameters: 23 parameters > 28 parameters.
- HERMES data are replaced and added deuteron target data sets.
- Different treatment for the normalization of the experiments.
- PDFs: MSTW2008.
- Relaxing some of the FFs assumptions.

$$D_{i}^{\pi^{+}}(z,Q_{0}) = \frac{N_{i}z^{\alpha_{i}}(1-z)^{\beta_{i}}[1+\gamma_{i}(1-z)^{\delta_{i}}]}{B[2+\alpha_{i},\beta_{i}+1]+\gamma_{i}B[2+\alpha_{i},\beta_{i}+\delta_{i}+1]}$$
$$D_{d+\bar{d}}^{\pi^{+}} = N_{d+\bar{d}}D_{u+\bar{u}}^{\pi^{+}} \qquad D_{\bar{u}}^{\pi^{+}} = D_{d}^{\pi^{+}}$$
$$D_{s}^{\pi^{+}} = D_{\bar{s}}^{\pi^{+}} = N_{s}z^{\alpha_{s}}D_{\bar{u}}^{\pi^{+}} \qquad \gamma_{c,b} \neq 0$$

Uncertanties

Goal: Provide Hessian sets to propagate FFs uncertainties.

HESSIAN METHOD

- Idea: Explore the vicinity of the best fit in quadratic approximation.
- Caveat: Quadratic approximation is not exactly what is used for the global fits, i.e. PDFs too.
- However, it is a good test of the convergence of the fitting procedure.



$$D_i^{\pi^+}(z, Q_0) = \frac{N_i z^{\alpha_i} (1-z)^{\beta_i} [1+\gamma_i (1-z)^{\delta_i}]}{B[2+\alpha_i, \beta_i+1] + \gamma_i B[2+\alpha_i, \beta_i+\delta_i+1]}$$

-New data to fit

- New data from PHENIX and STAR (Phys.Rev.C81(2010)064904; PRL 108(2012)072302;...).
- Data from the LHC (Phys.Lett.B717(2012)162;1307.1093;...).
- e+e- data from BELLE(1301.6183) and BaBar (1306.2895)
- SIDIS multiplicities from COMPASS (1307.3407).
- Final SIDIS multiplicities from HERMES (1212.5407).



(34)

ETSET

laBar

(10.54)

91.2 GeV

BELLE & BaBar

- BELLE and BaBar results can be fitted extremely well within the 68 and 90 % C.L.
- There is a drop on the large z regime for BELLE but it is consistent with the uncertainties.
- Large logarithmic corrections are expected at large values of z.
- Sensitive to a partial flavour separation



HERMES

- DSS cannot fit the new HERMES data for the smallest bin of z.
- In this new analysis, HERMES data have no problems to be fitted within the 68 and 90% C.L. for all bins of z.



COMPASS

- DSS also has some tensions with COMPASS data sets.
- For all values of z, COMPASS is well fitted.
- It is been shown also in the chi² ~ 1.01.



PHENIX & STAR

PDF uncertainties where computed with 90%CL MSTW and they are less significant than the scale ambiguities.



ALICE

- In the range of small pT, RHIC and LHC data showed a tension during the fitting.
- By introducing the cut on the pT, we achieved a reasonable agreement between both data sets.
- Nevertheless, we lost some data sets such as ALICE 900GeV which only stands with one point.
- Contribution of uncertainties due to PDF are again not relevant enough; the main contribution is coming from the scale variation.



Hessian method & convergence

 $D_i^{\pi^+}(z, Q_0^2) = N_i z^{\alpha_i} \left[(1-z)^{\beta_i} + \gamma_i (1-z)^{\delta_i} \right]$



parton-to-pion FFs

- Deviations from DSS is found on the gluon and charm FF.
- c-FF has a more flexible parametrisation (5 instead of 3 parameters).
- g-FF uncertainties is about 20% at 90%CL up to z > 0.5 and they increase towards larger values (Q = 10 GeV).



How good is the fit?

	DSS	NOW
Global	843/392(2.15)	1154.6/973(1.19)
LEP-SLAC	500.1/260(1.92)	412.6/260(1.58)
BELLE & BABAR		90.4/123(0.73)
HERMES	188.2/64(2.94)	175/128(1.36)
COMPASS		403.2/398(1.01)
RHIC	160.8/68(2.36)	45.7/53(0.86)
LHC		27.7/11(2.51)

-Conclusions-

- New methods for computing higher order corrections are needed for upcoming LHC observables.
- Mapping of momenta between real and virtual corrections permits to cancel soft and final-state collinear singularities.
- Fully local cancellation of IR and UV divergences through the LTD.
- LTD allows to build an algorithm for computing 4-dimensional representations of NLO cross sections.
- Extension of the LTD at NNLO and multi-leg processes is on the way.

-Conclusions-

- The numerical results shown that the breaking of the charge asymmetry parameter is very close to one.
- Tension between RHIC & LHC data have been avoided when a lower cut is introduced in the proton-proton collisions.
- The new data do not favor any symmetry violation.
- Uncertainties have been estimated using the standard iterative Hessian method.
- The analysis implemented strongly supports factorization and universality for the parton-to-pion FFs.

