Prof. Carlos Alberto López Andrade
Materia: Anillos y Campos

Tarea $\# 5$

1. Find all the units in $\mathbb{Z}[i]$.

2. If $a + bi$ is not a unit of $\mathbb{Z}[i]$ prove that $a^2 + b^2 > 1$.

3. Let $\pi \in \mathbb{Z}[i]$ be such that $d(\pi) = p$, where p is a prime in $\mathbb{Z}$. Show that π is a prime of $\mathbb{Z}[i]$.

4. Show that 2 is equal to the product of a unit and the square of a prime in $\mathbb{Z}[i]$.

5. Consider $\alpha = 7 + 2i$ and $\beta = 3 - 4i$ in $\mathbb{Z}[i]$. Find σ and ρ in $\mathbb{Z}[i]$ such that $\alpha = \beta\sigma + \rho$ with $d(\rho) < d(\beta)$.

6. Use a Euclidean algorithm in $\mathbb{Z}[i]$ to find a gcd of

   a) $8 + 6i$ and $5 - 15i$ in $\mathbb{Z}[i]$.
   b) $3 + 4i$ and $4 - 3i$ in $\mathbb{Z}[i]$.

Puebla, Pue., a 9 de marzo de 2014