of differential equations. A **system of ordinary differential equations** is two or more equations involving the derivatives of two or more unknown functions of a single independent variable. For example, if x and y denote dependent variables and t the independent variable, the following is a system of two first-order differential equations:

$$\begin{aligned}\frac{dx}{dt} &= 3x - 4y \\ \frac{dy}{dt} &= x + y.\end{aligned} \tag{4}$$

A solution of a system such as (4) is a pair of differentiable functions $x = \phi_1(t)$ and $y = \phi_2(t)$ that satisfies each equation in the system on some common interval I.

Remark

A few last words about implicit solutions of differential equations are in order. Unless it is important or convenient, there is usually no need to try to solve an implicit solution $G(x, y) = 0$ for y explicitly in terms of x. In Example 3 we can easily solve the relation $x^2 + y^2 - 4 = 0$ for y in terms of x to get two solutions, $y_1 = \sqrt{4 - x^2}$ and $y_2 = -\sqrt{4 - x^2}$, of the differential equation $dy/dx = -x/y$. But don't be misled by this one example. An implicit solution $G(x, y) = 0$ can define a perfectly good differentiable function ϕ that is a solution of a differential equation, but yet we may not be able to solve $G(x, y) = 0$ using analytical methods such as algebra. In Section 2.2 we shall see that $xe^{2y} - \sin xy + y^2 + c = 0$ is an implicit solution of a first-order differential equation. The task of solving this equation for y in terms of x presents more problems than just the drudgery of symbol pushing; *it can't be done.*

SECTION 1.1 EXERCISES

Answers to odd-numbered problems begin on page A-1.

In Problems 1–10 state whether the given differential equation is linear or nonlinear. Give the order of each equation.

1. $(1 - x)y'' - 4xy' + 5y = \cos x$
2. $x\dfrac{d^3y}{dx^3} - 2\left(\dfrac{dy}{dx}\right)^4 + y = 0$
3. $yy' + 2y = 1 + x^2$
4. $x^2\,dy + (y - xy - xe^x)\,dx = 0$
5. $x^3y^{(4)} - x^2y'' + 4xy' - 3y = 0$
6. $\dfrac{d^2y}{dx^2} + 9y = \sin y$
7. $\dfrac{dy}{dx} = \sqrt{1 + \left(\dfrac{d^2y}{dx^2}\right)^2}$
8. $\dfrac{d^2r}{dt^2} = -\dfrac{k}{r^2}$
9. $(\sin x)y''' - (\cos x)y' = 2$
10. $(1 - y^2)\,dx + x\,dy = 0$

In Problems 11–40 verify that the indicated function is a solution of the given differential equation. In some cases assume an appropriate interval