BENEMÉRITA UNIVERSIDAD AUTÓNOMA DE PUEBLA
FACULTAD DE CIENCIAS FÍSICO MATEMÁTICAS

Prof. Carlos Alberto López Andrade
Materia: Teoría de Grupos

Tarea # 9

I) List all the conjugate classes in S_3, find the $C(a)$'s, and verify the class equation.

II) List all the conjugate classes in S_4, find the $C(a)$'s, and verify the class equation.

III) Prove that

$$(1, 2, \ldots, r-1, r) = (2, 3, \ldots, r, 1) = (3, 4, \ldots, 1, 2) = \cdots = (r, 1, \ldots, r-2, r-1).$$

Conclude that there are exactly r such notations for this r-cycle.

IV) In S_n, prove that if $1 \leq r \leq n$, then there are $\frac{1}{r}\frac{n!}{(n-r)!}$ distinct r-cycles.

V) If in a finite group G an element a has exactly two conjugates, prove that G has a normal subgroup $N \neq (e), G$.

Using Theorem 2.11.2 (If $o(G) = p^n$ where p is a prime number, then $Z(G) \neq (e)$) as a tool, prove that:

VI) If $o(G) = p^n$, p a prime number and $G \neq (e)$, then G has a normal subgroup H of order p which is a subgroup of the center of G.

VII) If $o(G) = p^n$, p a prime number, then G has a subgroup of order p^α for all $0 \leq \alpha \leq n$.

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