***Prof. Carlos Alberto López Andrade***
***Materia: Teoría de Grupos***

# ***Tarea*** # 9

I) List all the conjugate classes in $S_3$, find the $C(a)$'s, and verify the class equation.

II) List all the conjugate classes in $S_4$, find the $C(a)$'s, and verify the class equation.

III) Prove that

$$(1, 2, \ldots, r-1, r) = (2, 3, \ldots, r, 1) = (3, 4, \ldots, 1, 2) = \cdots = (r, 1, \ldots, r-2, r-1).$$

Conclude that there are exactly $r$ such notations for this $r$-cycle.

IV) In $S_n$, prove that if $1 \leq r \leq n$, then there are $\frac{1}{r}\frac{n!}{(n-r)!}$ distinct $r$-cycles.

V) If in a finite group $G$ an element $a$ has exactly two conjugates, prove that $G$ has a normal subgroup $N \neq (e), G$.

Using Theorem 2.11.2 (If $o(G) = p^n$ where $p$ is a prime number, then $Z(G) \neq (e)$) as a tool, prove that:

VI) If $o(G) = p^n$, $p$ a prime number and $G \neq (e)$, then $G$ has a normal subgroup $H$ of order $p$ which is a subgroup of the center of $G$.

VII) If $o(G) = p^n$, $p$ a prime number, then $G$ has a subgroup of order $p^\alpha$ for all $0 \leq \alpha \leq n$.

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