(b) If $gug^{-1} \in U$ for all $g \in G$, $u \in U$, prove that $\hat{U}$ is a normal subgroup of $G$.

● 5. Let $U = \{xyx^{-1}y^{-1} \mid x, y \in G\}$. In this case $\hat{U}$ is usually written as $G'$ and is called the *commutator subgroup of* $G$.
(a) Prove that $G'$ is normal in $G$.
(b) Prove that $G/G'$ is abelian.
(c) If $G/N$ is abelian, prove that $N \supset G'$.
(d) Prove that if $H$ is a subgroup of $G$ and $H \supset G'$, then $H$ is normal in $G$.

● 6. If $N$, $M$ are normal subgroups of $G$, prove that $NM/M \approx N/N \cap M$.

● 7. Let $V$ be the set of real numbers, and for $a, b$ real, $a \neq 0$ let $\tau_{ab}: V \to V$ defined by $\tau_{ab}(x) = ax + b$. Let $G = \{\tau_{ab} \mid a, b \text{ real}, a \neq 0\}$ and let $N = \{\tau_{1b} \in G\}$. Prove that $N$ is a normal subgroup of $G$ and that $G/N \approx$ group of nonzero real numbers under multiplication.

8. Let $G$ be the dihedral group defined as the set of all formal symbols $x^i y^j$, $i = 0, 1$, $j = 0, 1, \ldots, n-1$, where $x^2 = e$, $y^n = e$, $xy = y^{-1}x$. Prove
(a) The subgroup $N = \{e, y, y^2, \ldots, y^{n-1}\}$ is normal in $G$.
(b) That $G/N \approx W$, where $W = \{1, -1\}$ is the group under the multiplication of the real numbers.

● 9. Prove that the center of a group is always a normal subgroup.

10. Prove that a group of order 9 is abelian.

11. If $G$ is a non-abelian group of order 6, prove that $G \approx S_3$.

● 12. If $G$ is abelian and if $N$ is any subgroup of $G$, prove that $G/N$ is abelian.

13. Let $G$ be the dihedral group defined in Problem 8. Find the center of $G$.

14. Let $G$ be as in Problem 13. Find $G'$, the commutator subgroup of $G$.

● 15. Let $G$ be the group of nonzero complex numbers under multiplication and let $N$ be the set of complex numbers of absolute value 1 (that is, $a + bi \in N$ if $a^2 + b^2 = 1$). Show that $G/N$ is isomorphic to the group of all positive real numbers under multiplication.

#16. Let $G$ be the group of all nonzero complex numbers under multiplication and let $\bar{G}$ be the group of all real $2 \times 2$ matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where not both $a$ and $b$ are 0, under matrix multiplication. Show that $G$ and $\bar{G}$ are isomorphic by exhibiting an isomorphism of $G$ onto $\bar{G}$.