for any integer n, in which case the factor group should suggest a relation to the integers mod n under addition. This type of relation will be clarified in the next section.

Problems

1. If H is a subgroup of G such that the product of two right cosets of H in G is again a right coset of H in G, prove that H is normal in G.
2. If G is a group and H is a subgroup of index 2 in G, prove that H is a normal subgroup of G.
3. If N is a normal subgroup of G and H is any subgroup of G, prove that NH is a subgroup of G.
4. Show that the intersection of two normal subgroups of G is a normal subgroup of G.
5. If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H.
6. Show that every subgroup of an abelian group is normal.

*7. Is the converse of Problem 6 true? If yes, prove it, if no, give an example of a non-abelian group all of whose subgroups are normal.

8. Give an example of a group G, subgroup H, and an element $a \in G$ such that $aHa^{-1} \subset H$ but $aHa^{-1} \neq H$.
9. Suppose H is the only subgroup of order $o(H)$ in the finite group G. Prove that H is a normal subgroup of G.
10. If H is a subgroup of G, let $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Prove
    (a) $N(H)$ is a subgroup of G.
    (b) H is normal in $N(H)$.
    (c) If H is a normal subgroup of the subgroup K in G, then $K \subset N(H)$ (that is, $N(H)$ is the largest subgroup of G in which H is normal).
    (d) H is normal in G if and only if $N(H) = G$.
11. If N and M are normal subgroups of G, prove that NM is also a normal subgroup of G.

*12. Suppose that N and M are two normal subgroups of G and that $N \cap M = (e)$. Show that for any $n \in N$, $m \in M$, $nm = mn$.

13. If a cyclic subgroup T of G is normal in G, then show that every subgroup of T is normal in G.

*14. Prove, by an example, that we can find three groups $E \subset F \subset G$, where E is normal in F, F is normal in G, but E is *not* normal in G.

15. If N is normal in G and $a \in G$ is of order $o(a)$, prove that the order, m, of Na in G/N is a divisor of $o(a)$.