6. Let a be a positive integer. What is the greatest common divisor of a and $a + 2$?

7. Show that if a and b are integers, not both 0, and c is a nonzero integer, then $(ca, cb) = |c|(a, b)$.

8. Show that if a and b are integers with $(a, b) = 1$, then $(a + b, a - b) = 1$ or 2.

9. What is $(a^2 + b^2, a + b)$, where a and b are relatively prime integers that are not both 0?

10. Show that if a and b are both even integers that are not both 0, then $(a, b) = 2(a/2, b/2)$.

11. Show that if a is an even integer and b is an odd integer, then $(a, b) = (a/2, b)$.

12. Show that if a, b, and c are integers such that $(a, b) = 1$ and $c \mid (a + b)$, then $(c, a) = (c, b) = 1$.

13. Show that if a, b, and c are mutually relatively prime nonzero integers, then $(a, bc) = (a, b)(a, c)$.

☞ 14. a) Show that if a, b, and c are integers with $(a, b) = (a, c) = 1$, then $(a, bc) = 1$.
b) Use mathematical induction to show that if $a_1, a_2, \ldots, a_n$ are integers, and b is another integer such that $(a_1, b) = (a_2, b) = \cdots = (a_n, b) = 1$, then $(a_1 a_2 \cdots a_n, b) = 1$.

15. Find a set of three integers that are mutually relatively prime, but any two of which are not relatively prime. Do not use examples from the text.

16. Find four integers that are mutually relatively prime such that any three of these integers are not mutually relatively prime.

17. Find the greatest common divisor of each of the following sets of integers.

a) 8, 10, 12
b) 5, 25, 75
c) 99, 9999, 0
d) 6, 15, 21
e) −7, 28, −35
f) 0, 0, 1001

18. Find three mutually relatively prime integers from among the integers 66, 105, 42, 70, and 165.

19. Show that if $a_1, a_2, \ldots, a_n$ are integers that are not all 0 and c is a positive integer, then $(ca_1, ca_2, \ldots, ca_n) = c(a_1, a_2 \ldots, a_n)$.

20. Show that the greatest common divisor of the integers $a_1, a_2, \ldots, a_n$, not all 0, is the least positive integer that is a linear combination of $a_1, a_2, \ldots, a_n$.

21. Show that if k is an integer, then the integers $6k - 1$, $6k + 1$, $6k + 2$, $6k + 3$, and $6k + 5$ are pairwise relatively prime.

22. Show that if k is a positive integer, then $3k + 2$ and $5k + 3$ are relatively prime.

23. Show that $8a + 3$ and $5a + 2$ are relatively prime for all integers a.

24. Show that if a and b are relatively prime integers, then $(a + 2b, 2a + b) = 1$ or 3.

25. Show that every positive integer greater than 6 is the sum of two relatively prime integers greater than 1.