

to date is that there are infinitely many integers n for which $n^2 + 1$ is either a prime or the product of two primes. This was shown by Henryk Iwaniec in 1973. Conjectures such as the $n^2 + 1$ conjecture may be easy to state, but are sometimes extremely difficult to resolve (see [Ri96] for more information).

3.2 Exercises

- 1. Find the smallest five consecutive composite integers.
- 2. Find one million consecutive composite integers.
- 3. Show that there are no “prime triplets,” that is, primes p , $p + 2$, and $p + 4$, other than 3, 5, and 7.
- 4. Find the smallest four sets of prime triplets of the form p , $p + 2$, $p + 6$.
- 5. Find the smallest four sets of prime triplets of the form p , $p + 4$, $p + 6$.
- 6. Find the smallest prime between n and $2n$ when n is
 - a) 3. c) 19.
 - b) 5. d) 31.
- 7. Find the smallest prime between n and $2n$ when n is
 - a) 4. c) 23.
 - b) 6. d) 47.

An unsettled conjecture asserts that for every positive integer n there is a prime between n^2 and $(n + 1)^2$.

- 8. Find the smallest prime between n^2 and $(n + 1)^2$ for all positive integers n with $n \leq 10$.
- 9. Find the smallest prime between n^2 and $(n + 1)^2$ for all positive integers n with $11 \leq n \leq 20$.
- 10. Verify Goldbach’s conjecture for each of the following values of n .
 - a) 50 c) 102 e) 200
 - b) 98 d) 144 f) 222

CHRISTIAN GOLDBACH (1690–1764) was born in Königsberg, Prussia (the city noted in mathematical circles for its famous bridge problem). He became professor of mathematics at the Imperial Academy of St. Petersburg in 1725. In 1728, Goldbach went to Moscow to tutor Tsarevich Peter II. In 1742, he entered the Russian Ministry of Foreign Affairs as a staff member. Goldbach is most noted for his correspondence with eminent mathematicians, in particular Leonhard Euler and Daniel Bernoulli. Besides his well-known conjectures that every even positive integer greater than 2 is the sum of two primes and that every odd positive integer greater than 5 is the sum of three primes, Goldbach made several notable contributions to analysis.