of validity for the solution. Where used, the symbols c_1 and c_2 denote constants.

11. $2y' + y = 0; \quad y = e^{-x/2}$

12. $y' + 4y = 32; \quad y = 8$

13. $\dfrac{dy}{dx} - 2y = e^{3x}; \quad y = e^{3x} + 10e^{2x}$

14. $\dfrac{dy}{dt} + 20y = 24; \quad y = \frac{6}{5} - \frac{6}{5}e^{-20t}$

15. $y' = 25 + y^2; \quad y = 5 \tan 5x$

16. $\dfrac{dy}{dx} = \sqrt{\dfrac{y}{x}}; \quad y = (\sqrt{x} + c_1)^2, x > 0, c_1 > 0$

17. $y' + y = \sin x; \quad y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$

18. $2xy\,dx + (x^2 + 2y)\,dy = 0; \quad x^2y + y^2 = c_1$

19. $x^2\,dy + 2xy\,dx = 0; \quad y = -\dfrac{1}{x^2}$

20. $(y')^3 + xy' = y; \quad y = x + 1$

21. $y = 2xy' + y(y')^2; \quad y^2 = c_1(x + \frac{1}{4}c_1)$

22. $y' = 2\sqrt{|y|}; \quad y = x|x|$

23. $y' - \dfrac{1}{x}y = 1; \quad y = x \ln x, x > 0$

24. $\dfrac{dP}{dt} = P(a - bP); \quad P = \dfrac{ac_1e^{at}}{1 + bc_1e^{at}}$

25. $\dfrac{dX}{dt} = (2 - X)(1 - X); \quad \ln\dfrac{2 - X}{1 - X} = t$

26. $y' + 2xy = 1; \quad y = e^{-x^2}\int_0^x e^{t^2}\,dt + c_1e^{-x^2}$

27. $(x^2 + y^2)\,dx + (x^2 - xy)\,dy = 0; \quad c_1(x + y)^2 = xe^{y/x}$

28. $y'' + y' - 12y = 0; \quad y = c_1e^{3x} + c_2e^{-4x}$

29. $y'' - 6y' + 13y = 0; \quad y = e^{3x}\cos 2x$

30. $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0; \quad y = e^{2x} + xe^{2x}$

31. $y'' = y; \quad y = \cosh x + \sinh x$

32. $y'' + 25y = 0; \quad y = c_1 \cos 5x$

33. $y'' + (y')^2 = 0; \quad y = \ln|x + c_1| + c_2$

34. $y'' + y = \tan x; \quad y = -\cos x \ln(\sec x + \tan x)$

35. $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0; \quad y = c_1 + c_2x^{-1}, x > 0$

36. $x^2y'' - xy' + 2y = 0; \quad y = x\cos(\ln x), x > 0$

37. $x^2y'' - 3xy' + 4y = 0; \quad y = x^2 + x^2 \ln x, x > 0$

38. $y''' - y'' + 9y' - 9y = 0; \quad y = c_1 \sin 3x + c_2 \cos 3x + 4e^x$

39. $y''' - 3y'' + 3y' - y = 0; \quad y = x^2e^x$

40. $x^3\dfrac{d^3y}{dx^3} + 2x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = 12x^2; \quad y = c_1x + c_2x\ln x + 4x^2, x > 0$

In Problems 41 and 42 verify that the indicated piecewise-defined function is a solution of the given differential equation.

41. $xy' - 2y = 0; \quad y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \ge 0 \end{cases}$