3. Find the quotient and remainder in the division algorithm with divisor 17 and dividend

   a) 100
   b) 289
   c) -44
   d) -100.

4. What can you conclude if a and b are nonzero integers such that $a \mid b$ and $b \mid a$?

5. Show that if a, b, c, and d are integers with a and c nonzero such that $a \mid b$ and $c \mid d$, then $ac \mid bd$.

6. Are there integers a, b, and c such that $a \mid bc$, but $a \nmid b$ and $a \nmid c$?

7. Show that if a, b, and $c \neq 0$ are integers, then $a \mid b$ if and only if $ac \mid bc$.

8. Show that if a and b are positive integers and $a \mid b$, then $a \leqslant b$.

9. Give another proof of the division algorithm by using the well-ordering property. (Hint: When dividing a by b, take as the remainder the least positive integer in the set of integers $a-qb$.)

10. Show that if a and b are odd positive integers, then there are integers s and t such that $a = bs + t$, where t is odd and $|t| < b$.

11. When the integer a is divided by the interger b where $b > 0$, the division algorithm gives a quotient of q and a remainder of r. Show that if $b \nmid a$, when $-a$ is divided by b, the division algorithm gives a quotient of $-(q+1)$ and a remainder of $b - r$, while if $b \mid a$, the quotient is $-q$ and the remainder is zero.

12. Show that if a, b, and c are integers with $b > 0$ and $c > 0$, such that when a is divided by b the quotient is q and the remainder is r, and when q is divided by c the quotient is t and the remainder is s, then when a is divided by bc, the quotient is t and the remainder is $bs + r$.

13. a) Extend the division algorithm by allowing negative divisors. In particular, show that whenever a and $b \neq 0$ are integers, there are integers q and r such that $a = bq + r$, where $0 \leqslant r < |b|$.

    b) Find the remainder when 17 is divided by -7.

14. Show that if a and b are positive integers, then there are integers q, r and $e = \pm 1$ such that $a = bq + er$ where $-b/2 < er \leqslant b/2$.

15. Show that if a and b are real numbers, then $[a+b] \geqslant [a] + [b]$.

16. Show that if a and b are positive real numbers, then $[ab] \geqslant [a][b]$. What is the corresponding inequality when both a and b are negative? When one is negative and the other positive?