47–51 Find a formula for the described function and state its domain.

47. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.

48. A rectangle has area 16 m^2. Express the perimeter of the rectangle as a function of the length of one of its sides.

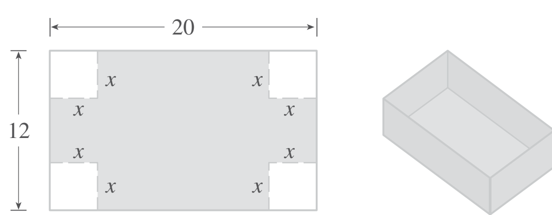
49. Express the area of an equilateral triangle as a function of the length of a side.

50. Express the surface area of a cube as a function of its volume.

51. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

52. A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 30 ft, express the area A of the window as a function of the width x of the window.

53. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



54. A taxi company charges two dollars for the first mile (or part of a mile) and 20 cents for each succeeding tenth of a mile (or part). Express the cost C (in dollars) of a ride as a function of the distance x traveled (in miles) for $0 < x < 2$, and sketch the graph of this function.

55. In a certain country, income tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$10,000 is taxed at a rate of 10%, up to an income of \$20,000. Any income over \$20,000 is taxed at 15%.

(a) Sketch the graph of the tax rate R as a function of the income I.

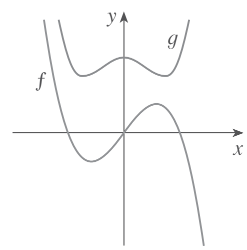
(b) How much tax is assessed on an income of \$14,000? On \$26,000?

(c) Sketch the graph of the total assessed tax T as a function of the income I.

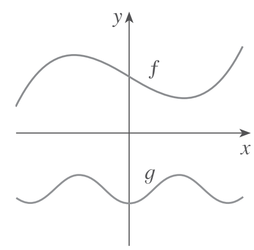
56. The functions in Example 10 and Exercises 54 and 55(a) are called *step functions* because their graphs look like stairs. Give two other examples of step functions that arise in everyday life.

57–58 Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

57.



58.



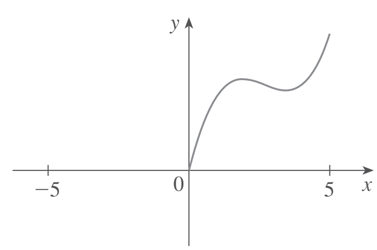
59. (a) If the point (5, 3) is on the graph of an even function, what other point must also be on the graph?

(b) If the point (5, 3) is on the graph of an odd function, what other point must also be on the graph?

60. A function f has domain $[-5, 5]$ and a portion of its graph is shown.

(a) Complete the graph of f if it is known that f is even.

(b) Complete the graph of f if it is known that f is odd.



61–66 Determine whether f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

61. $f(x) = x^{-2}$

62. $f(x) = x^{-3}$

63. $f(x) = x^2 + x$

64. $f(x) = x^4 - 4x^2$

65. $f(x) = x^3 - x$

66. $f(x) = 3x^3 + 2x^2 + 1$

EXAMPLE 14 Simplify the expression $\cos(\tan^{-1}x)$.

SOLUTION 1 Let $y = \tan^{-1}x$. Then $\tan y = x$ and $-\pi/2 < y < \pi/2$. We want to find $\cos y$ but, since $\tan y$ is known, it is easier to find $\sec y$ first:

$$\sec^2 y = 1 + \tan^2 y = 1 + x^2$$

$$\sec y = \sqrt{1 + x^2} \qquad \text{(since } \sec y > 0 \text{ for } -\pi/2 < y < \pi/2\text{)}$$

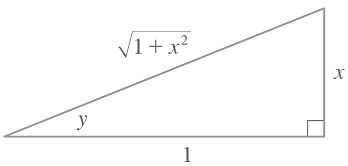


FIGURE 24

Thus
$$\cos(\tan^{-1}x) = \cos y = \frac{1}{\sec y} = \frac{1}{\sqrt{1 + x^2}}$$

SOLUTION 2 Instead of using trigonometric identities as in Solution 1, it is perhaps easier to use a diagram. If $y = \tan^{-1}x$, then $\tan y = x$, and we can read from Figure 24 (which illustrates the case $y > 0$) that

$$\cos(\tan^{-1}x) = \cos y = \frac{1}{\sqrt{1 + x^2}}$$

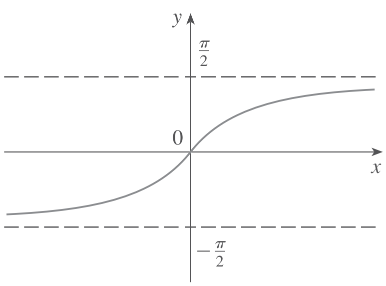


FIGURE 25
$y = \tan^{-1}x = \arctan x$

The inverse tangent function, $\tan^{-1} = \arctan$, has domain $\mathbb{R}$ and range $(-\pi/2, \pi/2)$. Its graph is shown in Figure 25.

We know that the lines $x = \pm\pi/2$ are vertical asymptotes of the graph of tan. Since the graph of $\tan^{-1}$ is obtained by reflecting the graph of the restricted tangent function about the line $y = x$, it follows that the lines $y = \pi/2$ and $y = -\pi/2$ are horizontal asymptotes of the graph of $\tan^{-1}$.

The remaining inverse trigonometric functions are not used as frequently and are summarized here.

11
$$y = \csc^{-1}x \ (|x| \geqslant 1) \iff \csc y = x \text{ and } y \in (0, \pi/2] \cup (\pi, 3\pi/2]$$
$$y = \sec^{-1}x \ (|x| \geqslant 1) \iff \sec y = x \text{ and } y \in [0, \pi/2) \cup [\pi, 3\pi/2)$$
$$y = \cot^{-1}x \ (x \in \mathbb{R}) \iff \cot y = x \text{ and } y \in (0, \pi)$$

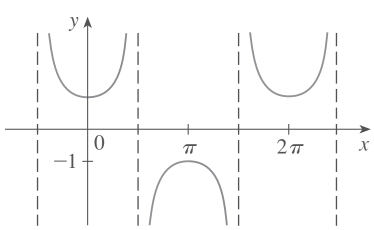


FIGURE 26
$y = \sec x$

The choice of intervals for y in the definitions of $\csc^{-1}$ and $\sec^{-1}$ is not universally agreed upon. For instance, some authors use $y \in [0, \pi/2) \cup (\pi/2, \pi]$ in the definition of $\sec^{-1}$. [You can see from the graph of the secant function in Figure 26 that both this choice and the one in (11) will work.]

1.6 Exercises

1. (a) What is a one-to-one function?
(b) How can you tell from the graph of a function whether it is one-to-one?

2. (a) Suppose f is a one-to-one function with domain A and range B. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?
(b) If you are given a formula for f, how do you find a formula for f^{-1}?
(c) If you are given the graph of f, how do you find the graph of f^{-1}?

3–14 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

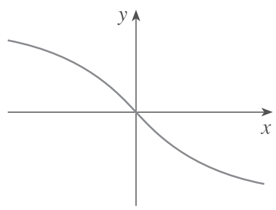
3.

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

4.

x	1	2	3	4	5	6
$f(x)$	1	2	4	8	16	32

5.



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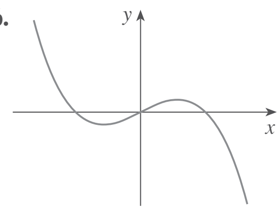

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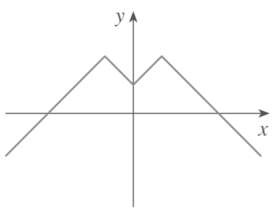

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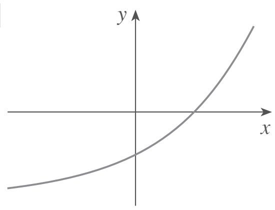


9. $f(x) = \frac{1}{2}(x + 5)$

10. $f(x) = 1 + 4x - x^2$

11. $g(x) = |x|$

12. $g(x) = \sqrt{x}$

13. $f(t)$ is the height of a football t seconds after kickoff.

14. $f(t)$ is your height at age t.

15–16 Use a graph to decide whether f is one-to-one.

15. $f(x) = x^3 - x$

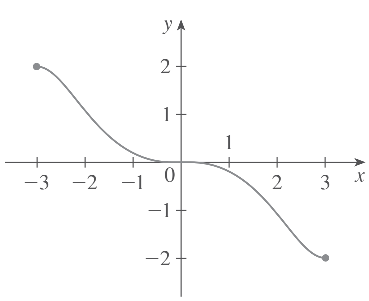
16. $f(x) = x^3 + x$

17. If f is a one-to-one function such that $f(2) = 9$, what is $f^{-1}(9)$?

18. Let $f(x) = 3 + x^2 + \tan(\pi x/2)$, where $-1 < x < 1$.
(a) Find $f^{-1}(3)$.
(b) Find $f(f^{-1}(5))$.

19. If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$.

20. The graph of f is given.
(a) Why is f one-to-one?
(b) State the domain and range of f^{-1}.
(c) Estimate the value of $f^{-1}(1)$.



21. The formula $C = \frac{5}{9}(F - 32)$, where $F \geqslant -459.67$, expresses the Celsius temperature C as a function of the Fahrenheit temperature F. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

22. In the theory of relativity, the mass of a particle with speed v is

$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Find the inverse function of f and explain its meaning.

23–28 Find a formula for the inverse of the function.

23. $f(x) = \sqrt{10 - 3x}$

24. $f(x) = \dfrac{4x - 1}{2x + 3}$

25. $f(x) = e^{x^3}$

26. $y = 2x^3 + 3$

27. $y = \ln(x + 3)$

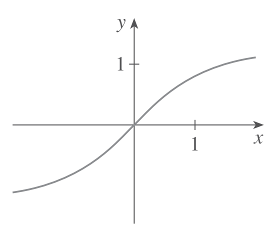
28. $y = \dfrac{1 + e^x}{1 - e^x}$

29–30 Find an explicit formula for f^{-1} and use it to graph f^{-1}, f, and the line $y = x$ on the same screen. To check your work, see whether the graphs of f and f^{-1} are reflections about the line.

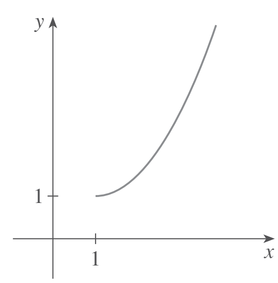
29. $f(x) = 1 - 2/x^2, \quad x > 0$

30. $f(x) = \sqrt{x^2 + 2x}, \quad x > 0$

31. Use the given graph of f to sketch the graph of f^{-1}.



32. Use the given graph of f to sketch the graphs of f^{-1} and $1/f$.



33. (a) How is the logarithmic function $y = \log_a x$ defined?
(b) What is the domain of this function?
(c) What is the range of this function?
(d) Sketch the general shape of the graph of the function $y = \log_a x$ if $a > 1$.