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Materia: Teoría de Grupos

Tarea # 10

- I) List all the conjugate classes in S_3 , find the $C(a)$'s, and verify the class equation.
- II) List all the conjugate classes in S_4 , find the $C(a)$'s, and verify the class equation.
- III) Prove that
- $$(1, 2, \dots, r-1, r) = (2, 3, \dots, r, 1) = (3, 4, \dots, 1, 2) = \dots = (r, 1, \dots, r-2, r-1).$$
- Conclude that there are exactly r such notations for this r -cycle.
- IV) In S_n , prove that if $1 \leq r \leq n$, then there are $\frac{1}{r} \frac{n!}{(n-r)!}$ distinct r -cycles.
- V) If in a finite group G an element a has exactly two conjugates, prove that G has a normal subgroup $N \neq (e), G$.
- Using Theorem 2.11.2 (If $o(G) = p^n$ where p is a prime number, then $Z(G) \neq (e)$) as a tool, prove that:
- VI) If $o(G) = p^n$, p a prime number and $G \neq (e)$, then G has a normal subgroup H of order p which is a subgroup of the center of G .
- VII) If $o(G) = p^n$, p a prime number, then G has a subgroup of order p^α for all $0 \leq \alpha \leq n$.

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