Cosmic Rays and the Search for a Violation of Lorentz Invariance

I. **Cosmic rays**

Phenomenology, GZK cutoff

II. **Violation of Lorentz Invariance**

Theory, maximal attainable velocities
Applications to decays, Čerenkov radiation
GZK cutoff and neutrino oscillation

III. **Cosmic γ-rays**

Radiation of blazars and γ-Ray-Bursts
(Photons in a non-commutative space)

IV. **Summary and Appendix** News from Pierre Auger Obs.

I. Cosmic Rays

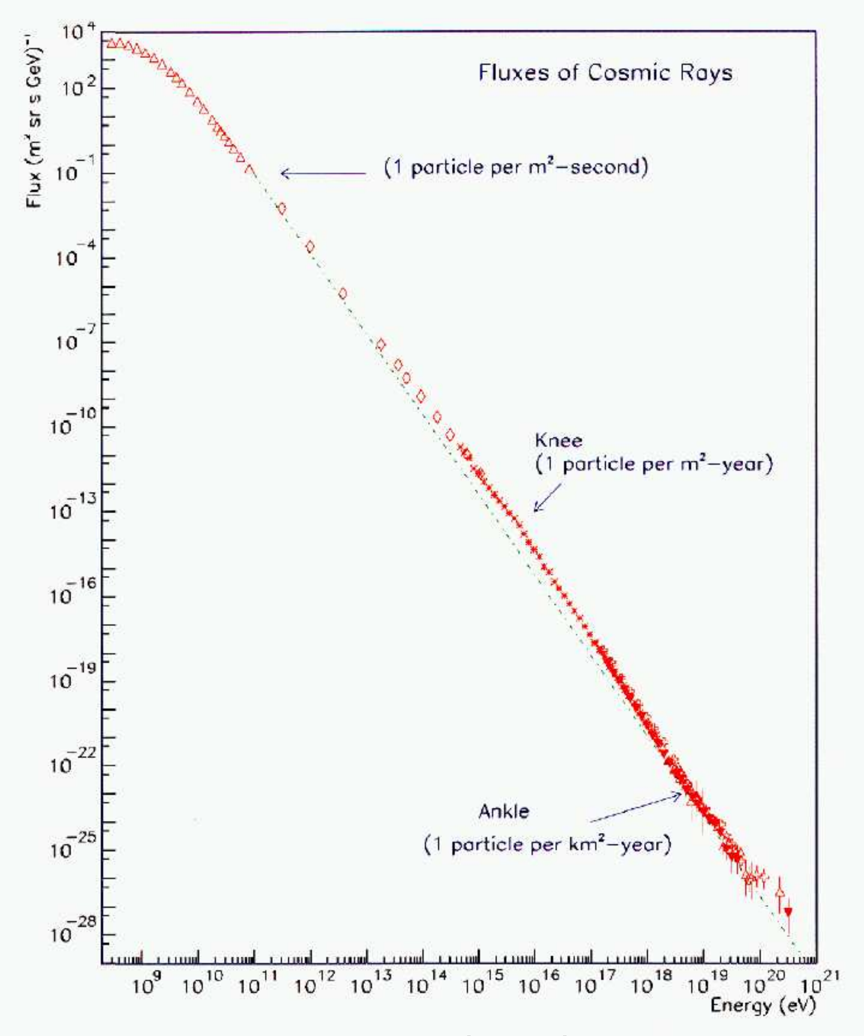
High energy particles and nuclei of cosmic origin

1912 Discovery by V.F. Hess with an electroscope on a balloon :
ionising radiation decreases up to 2000 m (radioactivity from earth)
but rises again above [Hess: → 5350 m; W. Kolhörster (1913/4) → 6300 m]
$\Rightarrow$ Cosmic origin (beyond sun)

1938 P. Auger : separated Geiger counters detect correlated events.
Extended Air Shower
Cascade triggered by a high energy primary particle. Estimate: $E \geq 10^{15}$ eV

Knowledge today: $\mathbf{E \approx 10^9 \ldots 10^{20}}$ **eV**
at high energy : ≈ 90 % protons, 9 % α, plus a few heavier nuclei; or Fe ions (?)

Isotropic; for charged rays source cannot be located (traditional picture)
[deflection by interstellar magnetic fields $\sim O(\mu\mathrm{G})$]



Flux of cosmic rays vs. energy, over broad interval essentially $\propto E^{-3}$

Origin ??? Historic proposal: Fermi mechanisms

Collisions in a magnetic cloud.

Later version : shock waves in gas of a supernova.

Explanation at best up to $\sim 10^{14}$ eV.

Predicts energy density $\propto E^{-2}$ [Observations close to E^{-3}]

Two classes of scenarios :

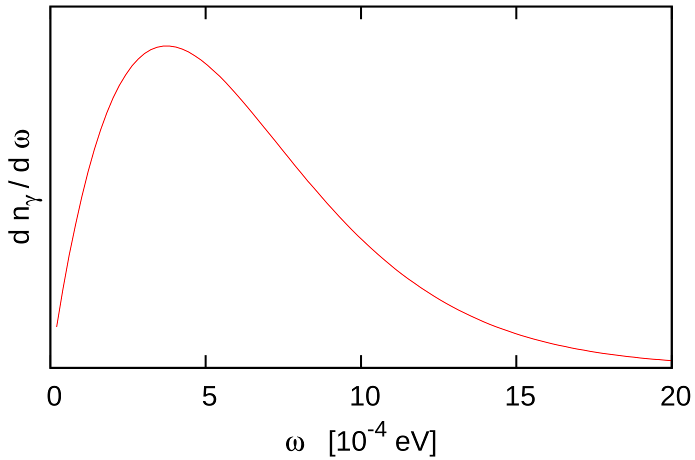
- "Bottom-up" : Sources build up huge energy
  (at least 7 orders of magnitude beyond man-made accelerators)
  Pulsars ? (rotating magnetised neutron stars) *Quasars ?* (→ Sect. 2)
  Active Galactic Nuclei ? (→ Appendix)

- "Top-down" : Decay of extremely heavy particles
  generated in Big Bang → energy available
  (magnetic monopoles, "wimpzillas" . . . ?? → high-E γ, ν flux, *not observed*)

1965 A. Penzias/R. Wilson discover Cosmic Microwave Background

Relic of the Big Bang, photons decoupled after $\approx 3.8 \cdot 10^5$ years

Very precise Planck distribution : $dn_\gamma/d\omega \propto \frac{\omega^2}{e^{\omega/kT}-1}$, ω : γ-energy

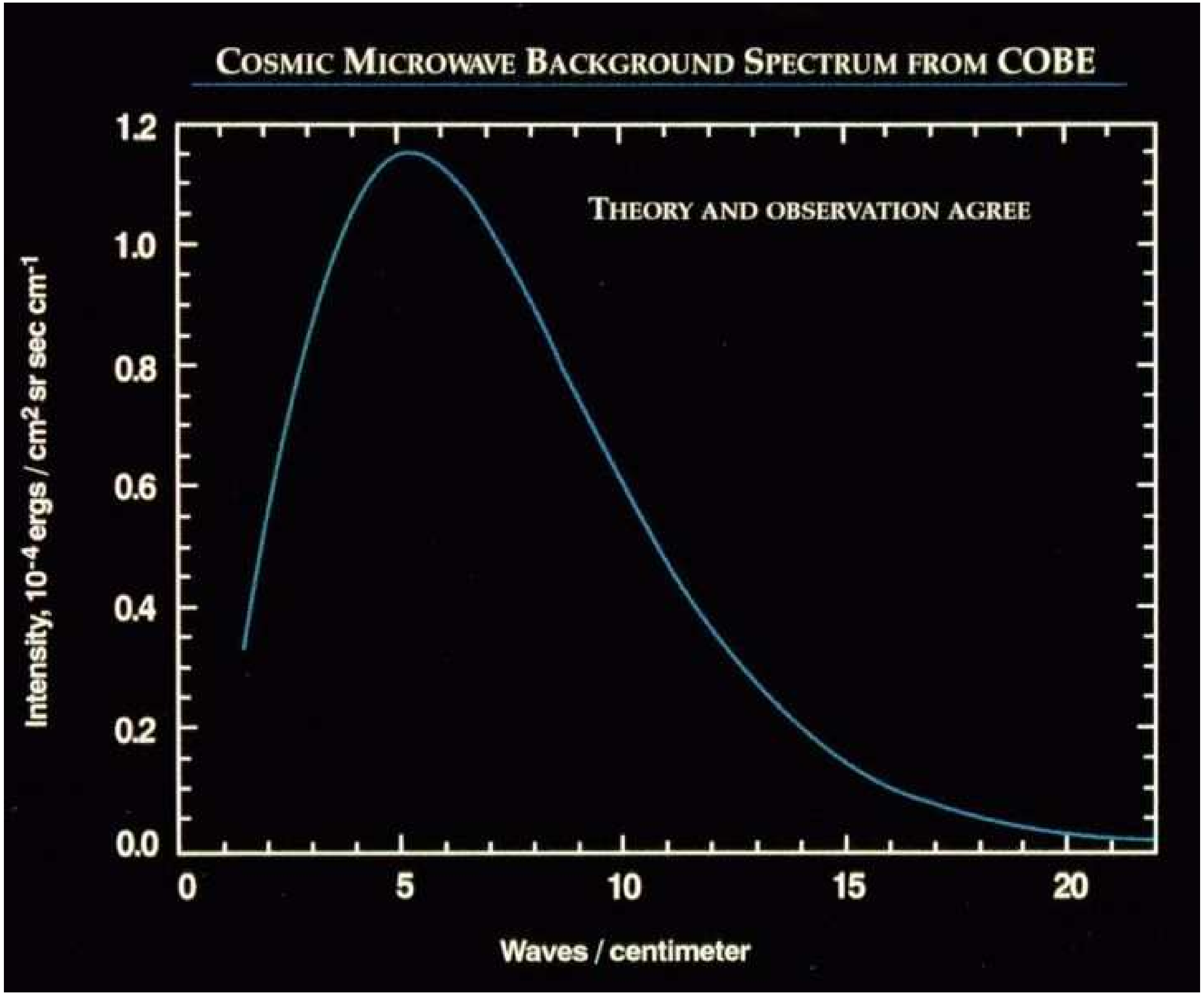


$\mathbf{T = 2.725(1)}$ **K**

$\int_0^\infty d\omega\, n_\gamma{}'(\omega) \simeq 411 \text{ cm}^{-3}$

$\langle\omega\rangle = \mathbf{6 \cdot 10^{-4}}$ **eV** ,

$\langle\lambda\rangle \simeq 1.9$ mm (microwaves)


COSMIC MICROWAVE BACKGROUND SPECTRUM FROM COBE
THEORY AND OBSERVATION AGREE
Intensity, 10⁻⁴ ergs / cm² sr sec cm⁻¹
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
Waves / centimeter

GZK Cutoff

1966 K. Greisen (Cornell), G.T. Zatsepin und V.A. Kuz'min (Lebedev)

Prediction: Cosmic rays have **"cutoff"** at $\underline{E \approx 6 \cdot 10^{19}\ \mathrm{eV}}$

Reason: Photopion production, in particular:

$$p + \gamma \to \Delta(1232\ \mathrm{MeV}) \left.\begin{array}{ll} \to & p + \pi^0 \\ \to & n + \pi^+ \ , \quad n \to p + e^- + \bar{\nu}_e \end{array}\right\} 99.4\%$$

[Further resonances: $\Delta(1620,\ 1700,\ \ldots)$, $p^*(1440,\ 1520\ \ldots)$ etc.
$\to\ p + \pi$ or $p + 2\pi$]

Threshold for proton energy: $E_p = E_0$

$$\begin{aligned} s &= (E_0 + \omega)^2 - (\vec{p}_p + \vec{p}_\gamma)^2 \qquad (\text{"laboratory", FRW metrics}) \\ &= \underbrace{E_0^2 - \vec{p}_p^{\,2}}_{m_p^2} + 2E_0\,\omega - 2\vec{p}_p\,\vec{p}_\gamma \simeq m_p^2 + \underbrace{4E_0\,\omega}_{\text{head}-\text{on}} \overset{!}{=} m_\Delta^2 \qquad (\text{rest frame of } \Delta) \end{aligned}$$

$$E_0 = \frac{m_\Delta^2 - m_p^2}{4\omega} \overbrace{=}^{\text{e.g. } \omega=5\langle\omega\rangle} \underline{6 \cdot 10^{19}\ \text{eV}}$$

Further kinemat. transformations → Inelasticity $K := \frac{\Delta E_p}{E_p} = \frac{1}{2}\left[1 - \frac{m_p^2 - m_\pi^2}{s}\right]$

Rest frame of the proton: $s = (m_p + \bar{\omega})^2 - \vec{p}_\gamma^{\,2} = m_p^2 + 2m_p\,\bar{\omega}$

Doppler effect: $\bar{\omega} = \gamma\omega(1 - \frac{v_p}{c}\cos\theta)$ (θ : scattering angle in "laboratory")

$\gamma = \frac{E_p}{m_p}$, e.g. $\frac{E_0}{m_p} \sim 10^{11}$ ⇒ $\langle\bar{\omega}\rangle_\theta \simeq 180\ \text{MeV} \cdot \frac{E_p}{E_0}$

$$K(\langle\bar{\omega}\rangle) = \frac{1}{2}\Big[1 - \frac{m_p^2 - m_\pi^2}{m_p(m_p + 2\langle\bar{\omega}\rangle)}\Big] = \begin{cases} 0.15 & \langle\bar{\omega}\rangle = 180\ \text{MeV} \\ 0.20 & \langle\bar{\omega}\rangle = 300\ \text{MeV}\ ,\ E_p = 2E_0 \end{cases}$$

τ : decay time of energy $E_p > E_0$ during journey through the Universe

$$\frac{1}{\tau(E_p)} = -\frac{kT}{2\pi^2\gamma^2}\int_{\bar{\omega}_0}^{\infty} d\bar{\omega}\,\sigma(\bar{\omega})K(\bar{\omega})\,\underbrace{\bar{\omega}\ln\left[1-e^{-\bar{\omega}/(2\gamma kT)}\right]}_{\text{from Planck distribution}}$$

(F.W. Stecker, ’68)

Cross-section σ is known from experiments:
proton at rest in γ radiation $\sigma \approx 0.1\ \mathrm{mb}$ (e.g. K.K. Wilson, ’58)

⇒ Computation of τ and corresp. path length $\ell \simeq \tau c \sim 10 \ldots 20\ \mathrm{Mpc}$

Minor corrections (additional photopion productions channels, discrete process . . .)

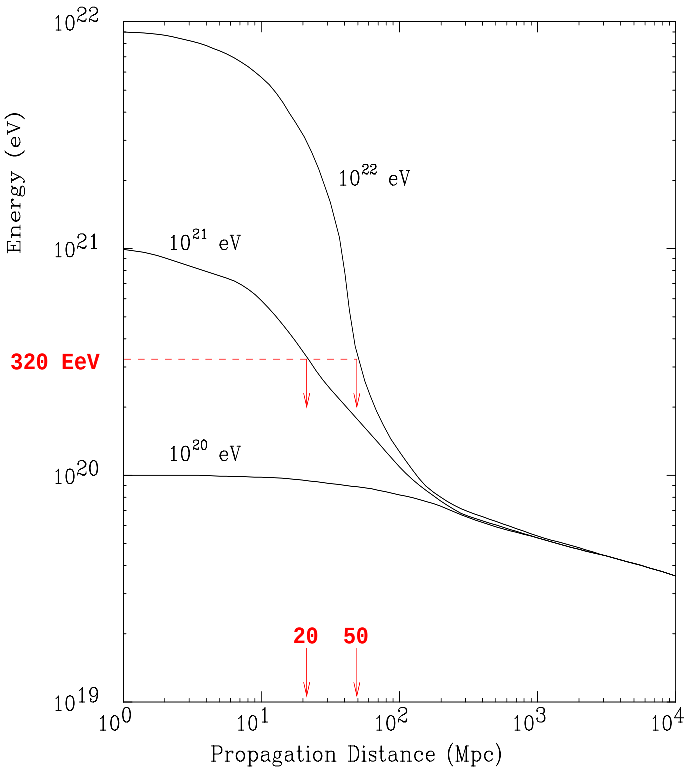
Heavy nuclei: photodisintegration → attenuation length even shorter

Also for protons: very high starting energy → energy loss more rapid

⇒ **Range with $\mathrm{E} >$ GZK cutoff is maximally $\sim 50 \ldots 100\ \mathrm{Mpc}$**

THE GZK CUTOFF

Energy at source



Energy attenuation of protons

Protons: photopion threshold @ ~50 EeV

Photons: pair production threshold @ ~200 TeV

Nuclei: photodisintegration above 50 EeV

Neutrinos: no problem!

For E>100 EeV, the source must be within ~50 Mpc

$[R_{\text{galactic plane}} \sim 15 \text{ kpc}] \ll$ **attenuation length** $\ll [R_{\text{visible Universe}} \sim 14 \text{ Gpc}]$

Source should be near-by (e.g. Virgo galaxy cluster, 20 Mpc).

Homogeneously distributed sources → pile-up at $E \lesssim E_0$
($5 \cdot 10^{17}$ eV $< E < E_0$: $p + \gamma \to p + e^+ + e^-$ but ΔE is small).

No sufficient acceleration mechanism is known, in particular not in our vicinity
→ Exceeding the GZK cutoff would be mysterious

Observations

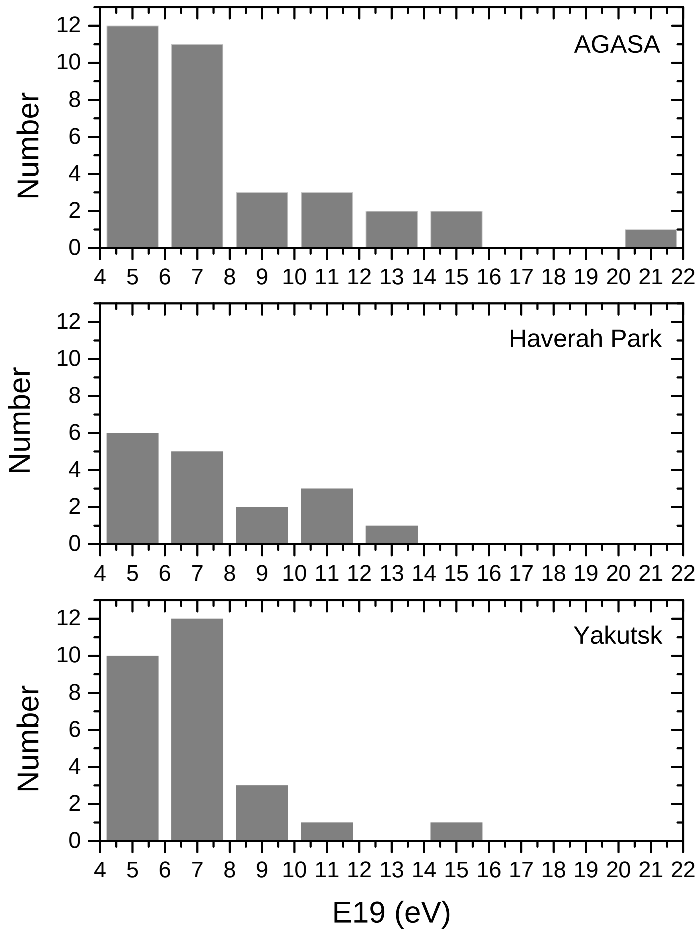
1963 J. Linsley et al. (New Mexico) one event at 10^{20} eV

1971 K. Suga et al. (Tokyo) new super-GZK event

1991 Fly's Eye (Utah) claims world record: $3 \cdot 10^{20}$ eV $(= 48 \text{ J})$

21st century : AGASA (Japan) numerous super-GZK events.
Spectrum agrees with Yakutsk (Russia) and Haverah Park (England),
but in contradiction to HiRes (Utah): seems to confirm cutoff.

De Marco/Blasi/Olinto (2003): Discrepancy might be explained statistically.



Super-GZK events at AGASA, Haverah Park and Yakutsk

Flux: $E > 10^{12}$ eV : ~ 10 primary particles / (m^2 min)
$E > 10^{18.5}$ eV : ~ 1 primary particle / (km^2 year)

Discrepancy between methods of detection ?

- AGASA etc. detect **air showers on surface of the earth**

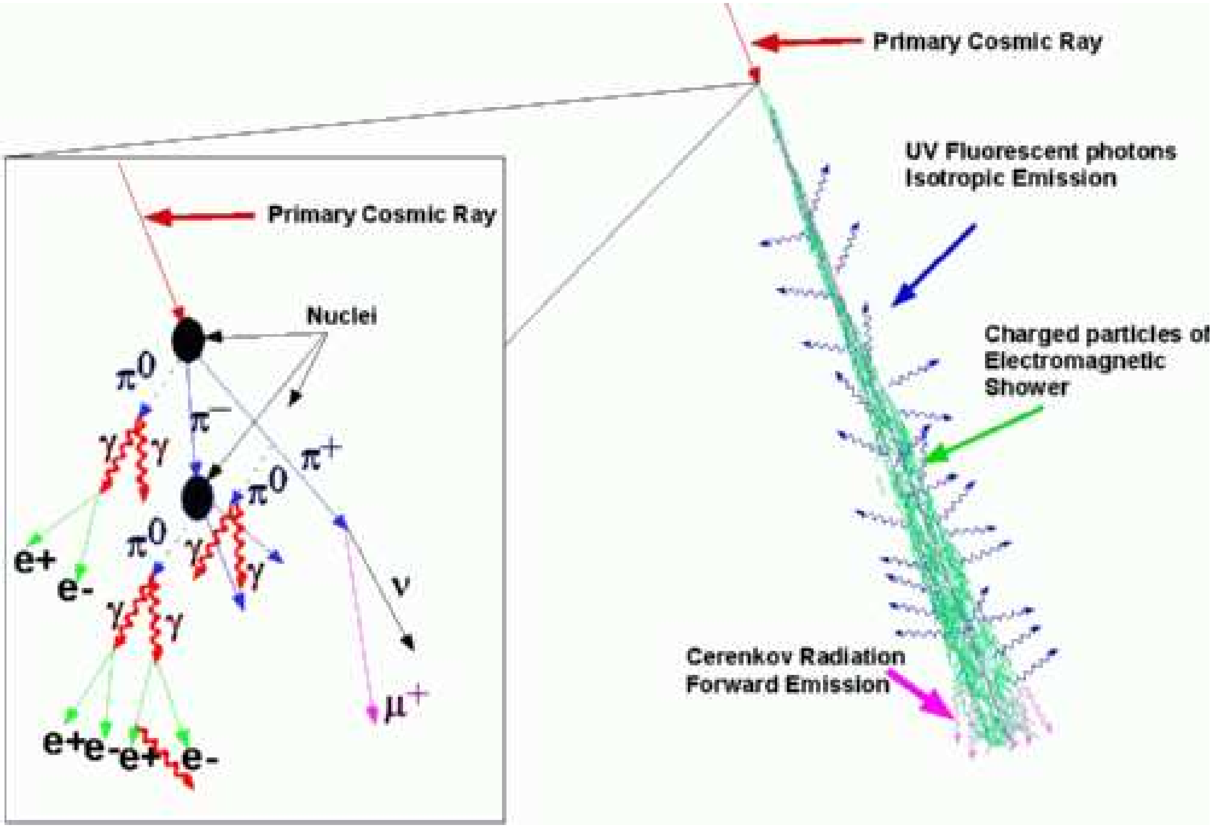
secondary: $\pi, K \cdots \to \mu \ldots$ (μ survive ~ 15 km to the earth).
$O(1)$ particle per GeV , up to 10^{11} particles
→ conclusions about energy of primary particle
{ shower is reconstructed with numerical methods, }

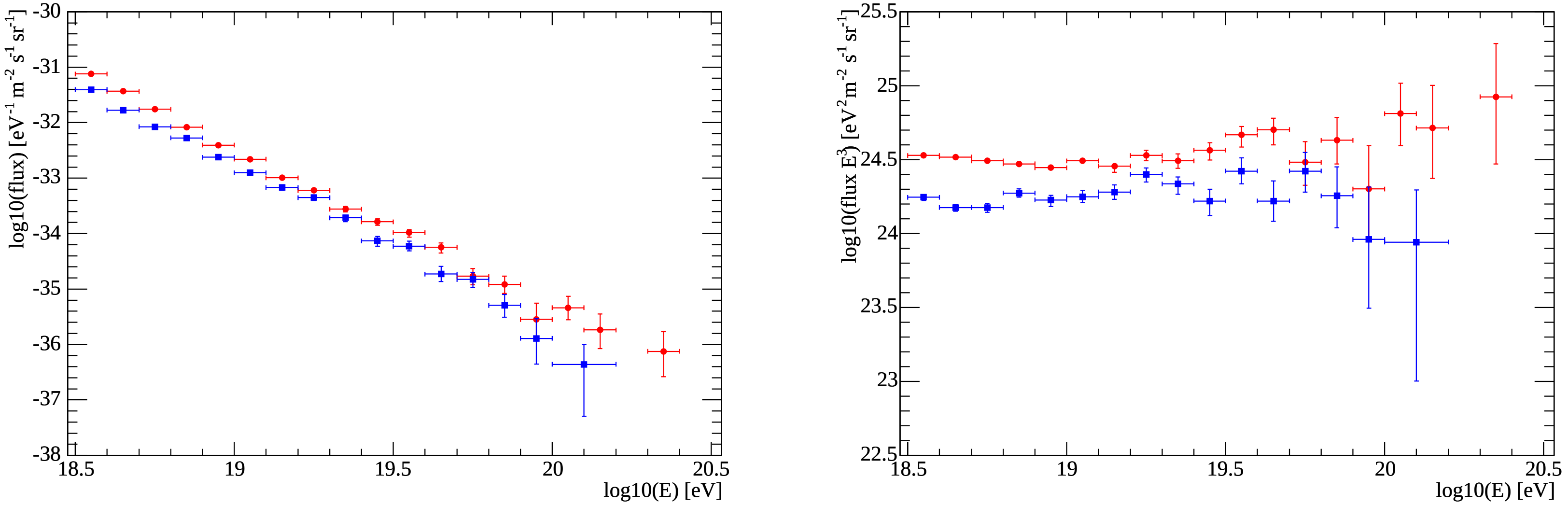
- HiRes: **Fluorescence : bluish/UV light emitted from excited N$_2$**

in nights without moon light and clouds visible by telescopes.

Heavy nuclei as primary particles → higher shower onset $\rightsquigarrow$ type of primary particle
[Record at $3 \cdot 10^{20}$ eV was presumably a heavy nucleus, e.g. oxygen]

Spallation : heavy nuclei break apart after a while (collision in gas clouds)
plus photodisintegration → high proton fraction hints at a **long path.**


Primary Cosmic Ray
Primary Cosmic Ray
Nuclei
π^0
π^-
π^+
γ
γ
π^0
π^0
γ
γ
γ
γ
e+
e-
ν
e+e-e+ e-
μ^+
UV Fluorescent photons
Isotropic Emission
Charged particles of
Electromagnetic
Shower
Cerenkov Radiation
Forward Emission

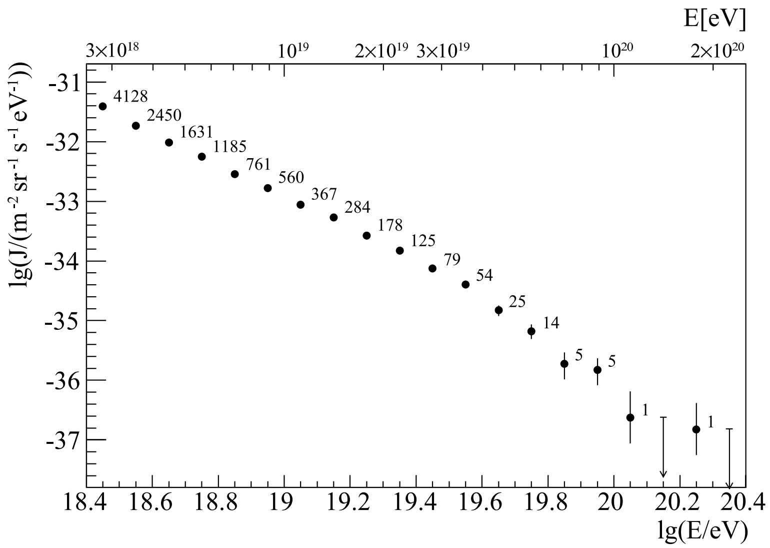


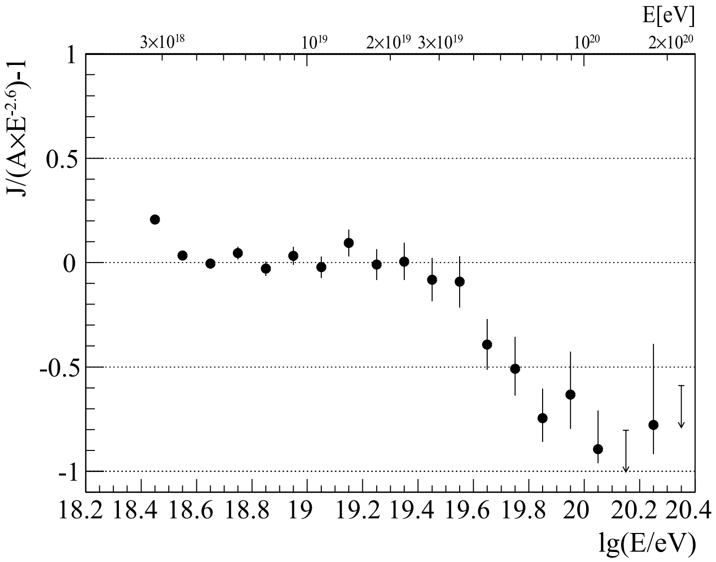
Data from **AGASA** vs. **HiRes**

Pierre Auger Project (in Argentina, operating in part since 2003, completed in 2008) combines both, hopes to resolve this issue

- on ground: water Čerenkov array
  1600 water tanks over 3000 km^2 (sizable statistics)
- Fluorescence: 24 telescopes verify correlation and energy calibration

$\rightarrow$ E to $\approx 22\%$ systematic error





Data by the Pierre Auger Collaboration (presented in July 2007 in Mexico)

Spectrum decays $\propto E^{-2.69(2)}$ between $E_{\text{ankle}} \simeq 10^{18.6}$ eV and $E_{\text{GZK}} \simeq 10^{19.6}$ eV

Clear reduction above E_{GZK} but some new super-GZK events...

Nov. 2007: Analysis and interpretation of arrival directions
→ Appendix

Not incompatilbe with overall flux $\propto E^{-3}$, space for speculations remains
E.g. violation of Lorentz symmetry (crucial for σ and K !) . . .

II. Lorentz Symmetry

So far assumed to hold, even at γ-factor $\sim 10^{11}$ (LEP probed up to $\gamma \sim 10^5$)
Central characteristic of relativity

- Special RT : holds **globally** [H.A. Lorentz (1904), H. Poincaré, A. Einstein (1905)]
- General RT : holds **locally** [A. Einstein ($\sim$ 1915)]

Field Φ (scalar, 4-vector, tensor, spinor) transforms in a representation D of the Lorentz group $SO(1,3)$,

$$\Phi(x) \ \rightarrow \ D(\Lambda)\Phi(\Lambda^{-1}x) \ , \qquad \Lambda \in SO(1,3) \ .$$

In particular scalars remain **Lorentz invariant (LI)**.

Theorem : { LI and Locality } $\Rightarrow$ **CPT Invariance**
W. Pauli, G. Lüders, R. Jost (1957)

Simultaneous charge conjugation (C), space reflection (P) and time inversion (T) .

O.W. Greenberg (2002): CPT violation $\Rightarrow$ **LI violation (LIV)** (not $\Leftarrow$)

[CPT tests with K_0 vs. $\bar{K}_0$: Δ mass (relative) $< 8 \cdot 10^{-19}$]

Direct tests of Lorentz sym. through invariance of c :

- Michelson-Morley type : $|\Delta c/c| \lesssim 10^{-11}$
- Atomic physics : precision tests of *specific LIV parameters*
  e.g. spin coupling of p , n, e^- to a possible "tensor background field" :
  rel. deviation $< 10^{-27}$
- Outlook: atomic clocks on ISS etc.

Impressive, **but:** CPT conserving LIV $\propto E^2$

assume e.g. on Planck scale $(M_{\text{Planck}} = 1/\sqrt{G} \approx 10^{28}\ \text{eV})$ $\text{LIV} \sim O(1)$
accelerators $E < 10^{13}\ \text{eV}$ $\rightarrow$ $\text{LIV} \sim O(10^{-30})$

on the other hand:
Laboratory LIV $\sim O(10^{-25})$, CPT violation $\propto E$ $\rightarrow$ at M_{Planck} : $\text{LIV} \sim O(10^{-10})$

$\rightarrow$ CPT **even** terms are more interesting.

Cosmic rays: hope for measuring effects not far below $M_{\rm Planck}$; long path !
Tests for theories like string, quantum gravity etc. are conceivable.
They like to install new fields in the vacuum, which may yield LIV.

Systematic Approach: (A. Kostelecký et al., since 1998)

"Standard Model Extension" , Lorentz sym. breaks spontaneously. Example:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - g\bar{\psi}\phi\psi - ig' G_{\mu\nu}\bar{\psi}\gamma^\mu\partial^\nu\psi + \dots$$

ϕ : Higgs field, SM: $m = g\langle\phi_0\rangle$
analogous: tensor field $\langle G_{00}\rangle > 0$, otherwise $\langle G_{\mu\nu}\rangle = 0$.
$\Rightarrow$ modified dispersion relation for each type of particle, depending on its coupling to $G_{\mu\nu}$

Kostelecký: > 100 parameters of this kind
preserve "all usual properties of the SM" (e.g. E, $\vec{p}$) except for LI (and CPT).
Special RT : Goldstone boson $\triangleq$ photon
General RT : various scenarios

Problem: why LIV at **high** energy ? Tiny g' : extreme hierarchy problem !

Pragmatic Approach (S. Coleman / S. Glashow '99)

$\mathcal{L}_{\rm eff}$ with explicit LIV parameters of mass dim. ≤ 4 (renormalisable),
CPT and gauge invariance persist, in addition $SO(3)$ sym. in a "preferred frame"

- Boson field $\vec{\Phi}$:

$$\mathcal{L} = \cdots + \frac{1}{2} \sum_{i=1}^{3} \partial_i \phi^a \varepsilon_{ab} \partial^i \phi^b \qquad (\varepsilon \ : \ \mathrm{sym.})$$

- Dirac spinor :

$$\mathcal{L} = \cdots + i \bar{\psi} \vec{\gamma} \vec{\partial} \, [\, \varepsilon_+ (1 + \gamma_5) + \varepsilon_- (1 - \gamma_5) \,] \, \psi$$

- Pure gauge terms, e.g. for $U(1)$: $E^i = F^{0i}$, $B^i = \frac{1}{2} \epsilon^{ijk} F_{jk}$
  rot'sym., ren'able terms : $\underbrace{\vec{E}^2 - \vec{B}^2 \quad , \quad \vec{E} \cdot \vec{B}}_{\rm LI} \quad , \quad \vec{B}^2 \quad , \quad \underbrace{\vec{A} \cdot \vec{B}}_{\rm breaks\ CPT}$
  → use also in YM theories : $\sum_a \vec{B}^a \cdot \vec{B}^a$ $\quad (a$: generators)

Leads to quasi-SM with 46 LIV parameters (many from *fermion generation mixing*)
with **gauge anomaly** $= 0$ (gauge invariance on quantum level)

Example: real scalar field with renormalised propagator

$$-iD^{-1} = (p^2 - m_0^2) f(p^2) + \varepsilon \vec{p}^{\,2} g(p^2)$$

[Minkowski space with $c = 1$, $p^2 = E^2 - \vec{p}^{\,2}$, m_0: renormalised at $\varepsilon \to 0$]
f, g : smooth functions with normalisation $f(m_0^2) = g(m_0^2) = 1$

LIV perturbation in $O(\varepsilon)$ shifts the poles to

$$\begin{aligned} E^2 &= \vec{p}^{\,2} + m_0^2 - \varepsilon \vec{p}^{\,2} \simeq \vec{p}^{\,2} c_{\mathrm{P}}^2 + m^2 c_{\mathrm{P}}^4 \\ \text{with} \quad & m = \frac{m_0}{1+\varepsilon} \quad , \quad c_{\mathrm{P}}^2 = 1 - \varepsilon \end{aligned}$$

Each particle receives its own **Maximal Attainable Velocity (MAV).**

[Group velocity : $\frac{\partial E}{\partial |\vec{p}|} = \frac{|\vec{p}|}{\sqrt{|\vec{p}|^2 + m^2 c_{\mathrm{P}}^2}} c_{\mathrm{P}}$]

Correction becomes significant when $\varepsilon \vec{p}^{\,2} / m_0^2 \sim O(1)$

$\Rightarrow$ tiny ε could be manifest at some tremendous energy ! (Hierarchy problem is back)

Applications:

- Decay at ultra high energy : particle$_0$ $\longrightarrow$ $\sum_a$ particle$_a$

(${m}$ negligible)

Decay condition:

$$\begin{aligned} c_0|\vec{p}_0| &= \sum_a c_a|\vec{p}_a| \geq c_{\min} \sum_a |\vec{p}_a| \geq c_{\min}|\vec{p}_0| \\ \Rightarrow &\quad c_0 \geq c_{\min} := \min_a c_a \end{aligned}$$

- Charged particle with $c_{\mathrm{P}}/c_\gamma = 1 + \varepsilon > 1$:

  "Vacuum Čerenkov radiation" at $v > c_\gamma$,

  i.e. $E > m/\sqrt{1 - c_\gamma^2/c_{\mathrm{P}}^2} \simeq m/\sqrt{2\varepsilon}$

▶ Protons survive $E \simeq 10^{20}\mathrm{eV}$ $\Rightarrow$ $\varepsilon_p < \frac{m_p^2}{2E^2} \approx 5 \cdot 10^{-23}$

better than bound from atomic physics (but only upper bound)

▶ Cosmic e$^\pm$ observed up to $E \simeq 1\ \mathrm{TeV}$ $\Rightarrow$ $\varepsilon_e < 10^{-13}$

- GZK Cutoff

Consider head-on collision $p+\gamma \rightarrow \Delta(1232)$ with $\mathbf{c_\gamma = c_\Delta = 1 \; , \; c_p = 1 - \varepsilon}$

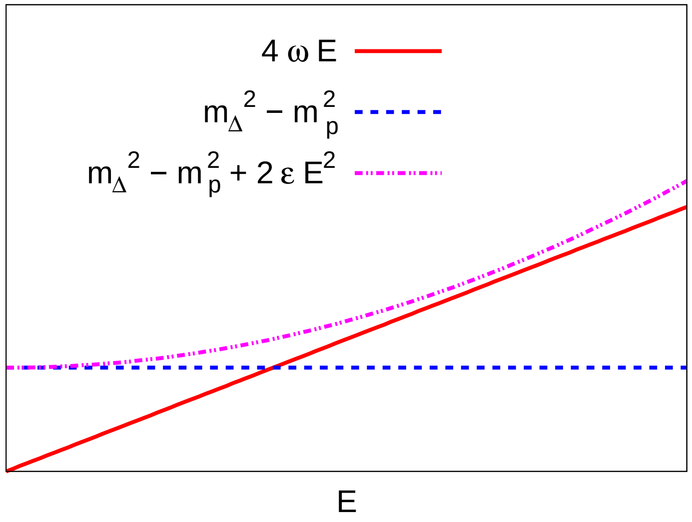
Constraint for a Δ resonance :

$$m_\Delta^2 < (E+\omega)^2 - (p_i - \omega)^2 \simeq \underbrace{E^2 - p_i^2} + 2\omega(E + p_i)$$

E, p_i for a proton in the "laboratory" :

$$E^2 - p_i^2 (1-\varepsilon)^2 = m_p^2 (1-\varepsilon)^4 \underbrace{\longrightarrow}_{E \gg m_p \, , \, |\varepsilon| \ll 1} \underbrace{E^2 - p_i^2} \simeq m_p^2 - 2\,\varepsilon\, E^2$$

$$\Rightarrow \quad \underline{m_\Delta^2 - m_p^2 + \mathbf{2\,\varepsilon\,E^2} < 4\omega E} \quad \text{avoids photopion production}$$



$$m_\Delta^2 - m_p^2 + 2\,\varepsilon\,E^2 < 4\omega E$$

- At $\varepsilon = 0$: minimal energy $E_0 = \frac{m_\Delta^2 - m_p^2}{4\omega}$
- With ε included, only soluble if

$$\varepsilon < \frac{\omega}{2E_0} \simeq \frac{2\omega^2}{m_\Delta^2 - m_p^2}\Big|_{\omega=6\cdot10^{-4}\ \mathrm{eV}} = 1.9 \cdot 10^{-25}$$

A tiny ε could remove the GZK cutoff !

[For slow protons the resonance $p + \gamma \to \Delta$ persists.]

This rules out the Δ channel for the photopion production.

Next candidate : $p + \gamma \to p^*(1435) \to p + \pi$

at ultra high energy : decay only for $c_\pi - c_p < 5 \cdot 10^{-24}$
we could close this channel too . . .

Farrar/Biermann (1998) :
the 5 top events $(> 10^{20}\ \mathrm{eV})$ all originate from the direction of a quasar.

[Quasi-stellar radio source: extremely bright centre of a young galaxy]

Coleman/Glashow : primary particle of the super-GZK events could be **neutrons** :

- $c_n < c_p$: no β-decay at high energy
- $c_n < c_\Delta$: protected from the GZK cutoff
- hardly deflected by magnetic fields

[Today (with $O(100)$ super-GZK events) quasar hypothesis out of fashion,
but clustering of directions revitalised, neutral primary particles (?)]

Maximal Attainable Velocities of the Neutrinos

Three bases for the neutrino states:
eigenstates of flavour, of mass m_0 or of **MAV** c_ν.

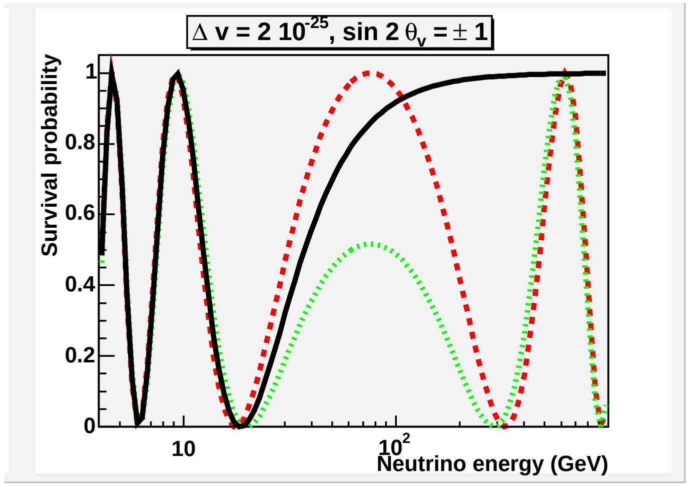
In principle neutrino oscillation is possible even at $m_\nu = 0$, but not compatible with phen. data.
(Lipari/Lusignoli '99)

We concentrate on the oscillation $\nu_\mu \leftrightarrow \nu_\tau$.
Assumption: dominant effect due to flavour-mixing of the mass states,
plus ev. sub-dominant effect from

$$\begin{aligned} \Delta v &= \mathrm{MAV}(\nu_1) - \mathrm{MAV}(\nu_2) \\ \theta_v &= \text{mixing angle of } |\nu_\mu\rangle \text{ and } |\nu_\tau\rangle \text{ in MAV basis} . \end{aligned}$$

Δv and θ_v modify the life time of ν_μ .

Example of the MACRO Collaboration (Gran Sasso) :



Survival probability of ν_μ over 10 000 km at $\Delta v = 2 \cdot 10^{-25}$, $\sin 2\theta_v = 0$, 1 , -1 .

Sensitivity at high energy of the ν_μ.

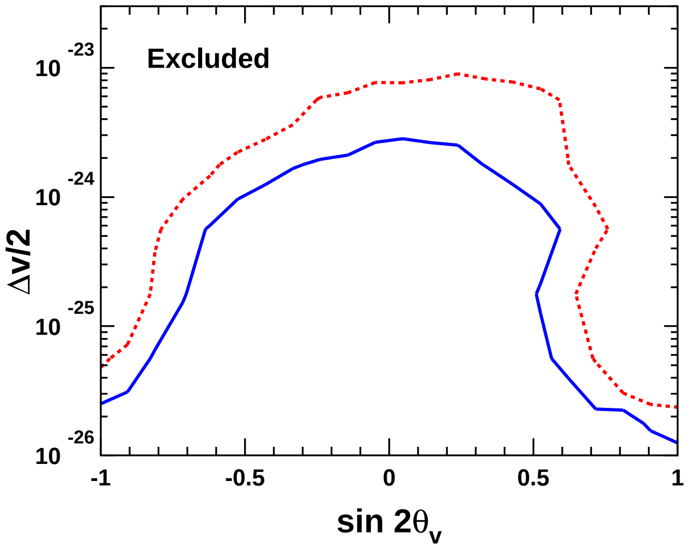
Consider a cosmic ν with $O(100)$ GeV , for $m_\nu \lesssim 1$ eV : $\gamma \gtrsim 10^{11}$ (like proton).

Detection of upward directed μ from $\nu_\mu + N \to \mu + \ldots$

multi-Coulomb scattering $\longrightarrow$ reconstruction of E_μ and E_{ν_μ} ,

58 events with $E_{\nu_\mu} > 130$ GeV, compare to flux at low E_{ν_μ}

Result (2004) :



Variation of Δv and θ_v does not improve the fit.

For arbitrary θ_v : $|\Delta v| < 6 \cdot 10^{-24}$ (90 % C.L.)

[Agreement with Super-Kamiokande K2K data (Fogli et al. '99)]

III. Cosmic γ-rays

We now consider the photons themselves (so far in the background).

Highest energy $E_\gamma > 50\ \mathrm{TeV}$ from Crab nebula (rest of a supernova, distance : 2 kpc).

Strongest sources *beyond* our galaxy :
Blazars, e.g. Markarian 501 (HEGRA, 1999), $E_\gamma \approx 20\ \mathrm{TeV}$,
distance 157 Mpc (from redshift).

Subset of "Active Galactic Nuclei", environment of a super-massive Black Hole,
driven by swallowed matter → emits $\gamma, e^\pm \ldots$
A few hundreds are known, here distance and direction can often be determined.

New puzzle similar to GZK

We expect pair creation with IR background photons

$$\gamma_{\mathrm{UV}}(E) + \gamma_{\mathrm{IR}}(\omega) \ \rightarrow\ e^+ + e^- \ldots [\mathrm{Compton\ scattering}]^{-1} \rightarrow \mathrm{cascade}$$

In centre-of-mass system: $\bar\omega = E/\gamma = \gamma\omega \ \rightarrow$ condition: $\bar\omega^2 = E\omega > m_e^2$.

Example: for $E \sim 10 \ \mathrm{TeV}$ scattering at $\omega \gtrsim 3 \ \mathrm{meV}$.

Despite the low density, this cross-section σ is sufficient,
to practically exclude E_γ over such long distances.

Stecker/Glashow '01 : Way out analogous to GZK

$$c_e = c_\gamma + \varepsilon$$

Condition for head-on collisions : $\quad 2E\omega - E^2\varepsilon > 2m_e^2$

$\varepsilon > 0$ could increase the energy threshold, or avoid pair creation completely
$\Rightarrow$ Universe becomes transparent for all photons.

No pair creation for $\varepsilon \geq \frac{2}{E^2}(E\omega - m_e^2)|_{E=20 \ \mathrm{TeV}, \ \omega=0.003 \ \mathrm{eV}} = 2 \cdot 10^{-15}$

below bound for vacuum Čerenkov radiation of the electron, $\varepsilon < 10^{-13}$.

However: little known radio background could resolve puzzle

γ-Ray-Bursts (GRB)

Emitted in powerful energy eruptions for short periods (sec. to min.), temporarily brightest γ source in the sky.

Sources are small, merger of Neutron Stars or Black Holes or . . .

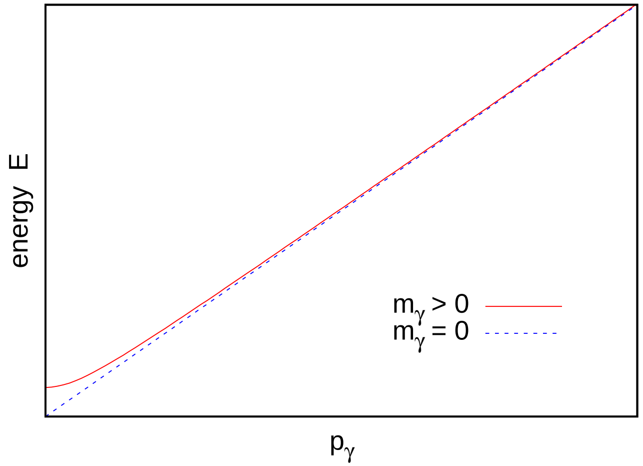
Known since 1973, homogeneous distribution, $E_\gamma = 10^4 \ldots 10^8 \text{ eV}$

Discovery from satellites, redshift measured from ground.
Direct identification from ground more efficient (La Silla, Chile).

2005: Observation from 4 Gpc, i.e. from Early Universe ($< 10^9$ y).

Amelino-Camelia et al. : test for dispersion relation: $\underline{v_\gamma(E) = \text{const.}}$?

Simplest attempt : $\mathbf{m_\gamma > 0}$, $v_\gamma = \frac{\partial E}{\partial p} \neq \text{const.}$



Almost same time of arrival after a long journey → $m_\gamma < 2.4 \cdot 10^{-11}$ eV (Schaefer, '99)

However :
much better bound from laboratory $m_\gamma < 6 \cdot 10^{-17}$ eV (CERN Data Booklet)

We stay with $m_\gamma = 0$.

Here Coleman/Glashow ansatz $E^2 = p^2 c_\gamma^2$ does not help.

- “Doubly Special Relativity”:

Class of theoretical approaches, which try to introduce a second absolute bound, in addition to c (Galilei: 0, Einstein: 1, are there more ?)

Example: H.S. Snyder (1947): absolute minimal length (maybe Planck length $1/M_{\rm Planck} \simeq 10^{-35}$ m)

Idea: proceed as with angular momentum operator L_3 from a 5d perspective ($c = 1$).

$$S = x_0^2 - x_1^2 - x_2^2 - x_3^2 - \mathbf{x_4^2}$$

$S = a^2$: 4d de Sitter space inside the 5d light cone.

Generation of transformations, which leave S invariant:

$$\begin{array}{llll} L_3 = \frac{\hbar}{i}(x_1\partial_2 - x_2\partial_1) & \text{invariant} & x_1^2 + x_2^2, \ x_0, \ x_3, \ x_4 & \\ X = \frac{a}{i}(x_1\partial_4 - x_4\partial_1) & \dots & x_1^2 + x_4^2, \dots & \text{(4d LIV)} \\ T = ai(x_0\partial_4 + x_4\partial_0) & \dots & x_0^2 - x_4^2, \dots & \text{(4d LIV)} \end{array}$$

Spectrum of X is discrete

$X\psi = \lambda\psi$, $\begin{pmatrix} x_1 \\ x_4 \end{pmatrix} = r \begin{pmatrix} \sin\varphi \\ \cos\varphi \end{pmatrix}$, $X = \frac{a}{i}\partial_\varphi$, $\psi \propto \exp(\frac{i}{a}\varphi\lambda)$

$\psi(\varphi) = \psi(\varphi + 2\pi) \qquad \Rightarrow \lambda = \frac{n}{a}$, $n \in \mathbb{Z}$

Position operators do not commute: $[X, Y] = \frac{ia}{\hbar} L_3$ etc.

→ new uncertainty relation $\min(\Delta X \ \Delta Y) \propto a^2$

Minimal length a as an absolute constant, 4d non-locality, but 5d LI.

Interpretation as event horizon in a mini Black Hole
matches $a =$ Planck length (Doplicher/Fredenhagen/Roberts '95)

Currently popular version: commutators as constant "tensor field"

$$[X_\mu, X_\nu] = i\Theta_{\mu\nu}$$

observer independent, sets min. area (tensor under deformed Lorentz trafo).

Non-commutativity affects pure $U(1)$ gauge field :

picks up a YM-type self-coupling $\longrightarrow$ deformed photon dispersion

1-loop result takes the form (Matusis/Susskind/Toumbas '00)

$$E^2 = \vec{p}^{\,2} + \frac{C}{(p\Theta)^2}$$

[on quantum level the new UV term $\Theta_{\mu\nu}$ causes also IR divergence (additional uncertainty !)]

▶ Amelino-Camelina et al. (2003) :

Analysis of GRB radiation $\Rightarrow$ $\|\Theta\| > 10^{-40}\ \mathrm{cm}^2$, otherwise effect should be *larger*.

However:

- $\Theta = 0$ is not excluded in this way.
- $C < 0$ i.e. 1-loop result is actually IR unstable ! (Landsteiner/Lopez/Tytgat '01).
  (SUSY cancels IR divergence . . .)

NC QED revisited non-perturbatively (W.B./Nishimura/Susaki/Volkholz '06)

- Commutative plane (x_3, x_4) → Lattice

includes Euclidean time (enables transition to Minkowski signature)

- NC plane $(\hat{x}_1, \hat{x}_2)$, $[\hat{x}_1, \hat{x}_2] = i\theta$

Lattice structure : $\exp\left(i\frac{2\pi}{a}\hat{x}_\mu\right) = \hat{1\!\!1} \quad (\mu = 1, 2)$

Momenta commute, usual periodicity

$$\begin{aligned} e^{ik_\mu \hat{x}_\mu} &= e^{i(k_\mu + \frac{2\pi}{a})\hat{x}_\mu} \\ \hat{1\!\!1} &= e^{i(k_\mu + \frac{2\pi}{a})\hat{x}_\mu}\, e^{-ik_\nu \hat{x}_\nu} = \dots = \hat{1\!\!1} \exp\left(\frac{i\pi}{a}\theta(k_2 - k_1)\right) \\ \Rightarrow \quad & \frac{\theta}{2a} k_\mu \in \mathbb{Z} \quad : \ \text{momenta discrete, lattice periodic} \end{aligned}$$

Periodic $N \times N$ lattice: $k_\mu = \frac{2\pi}{aN} n_\mu \quad (n_\mu \in \mathbb{Z}) \quad \Rightarrow \quad \underline{\theta = \frac{1}{\pi}\mathbf{Na^2}}$

Double Scaling Limit : $\left.\begin{array}{r}\text{continuum }\ a \to 0 \\ \text{infinite volume }\ Na \to \infty\end{array}\right\} Na^2 = const.$

Simultaneous UV and IR limit, which keeps $\theta = const.$ (Szabo '01)

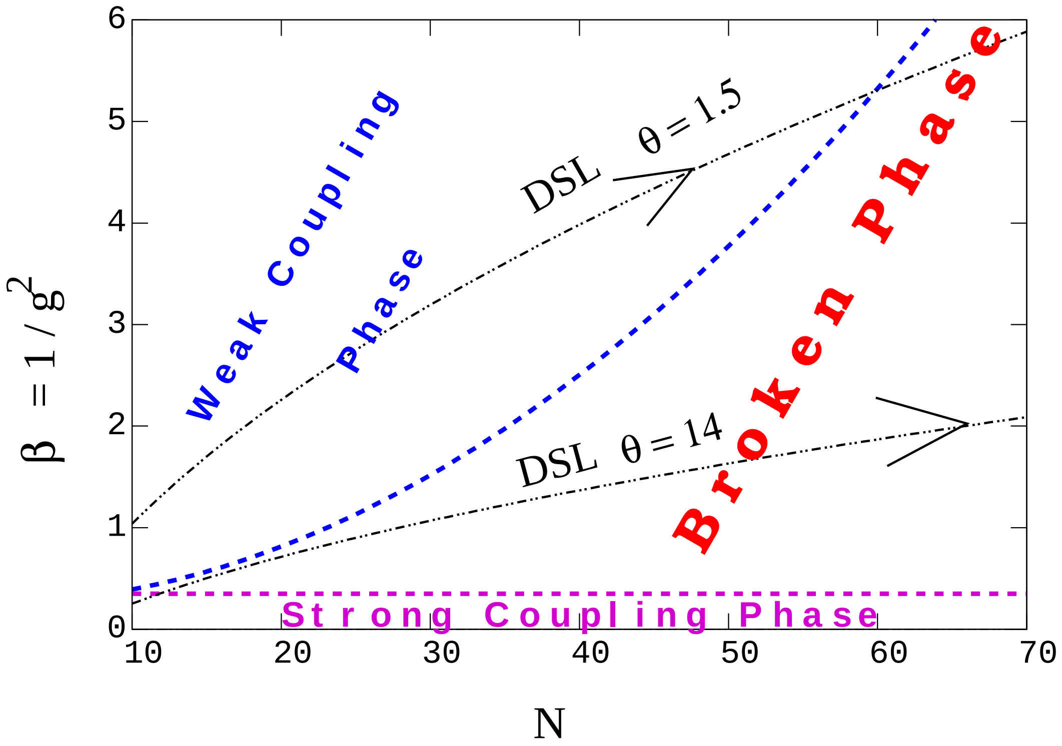
$U(1)$ gauge theory on a NC lattice can be mapped onto a
"twisted Eguchi-Kawai model" ($U(N)$ matrices in one point) (Ambjørn et al. '01)
→ numerically tractable

Yang-Mills type self-interaction and gauge transformations are **non-local** on scale $\sqrt{|\theta|}$.

In this range: gauge invariant open Wilson lines
carry momentum → order parameters for spont. breaking of transl. sym.

Numerical observation:
Double Scaling Limit $\beta \equiv \frac{1}{g^2} \propto \sqrt{N}$ stabilises a variety of observables ($a = 1/\beta$)

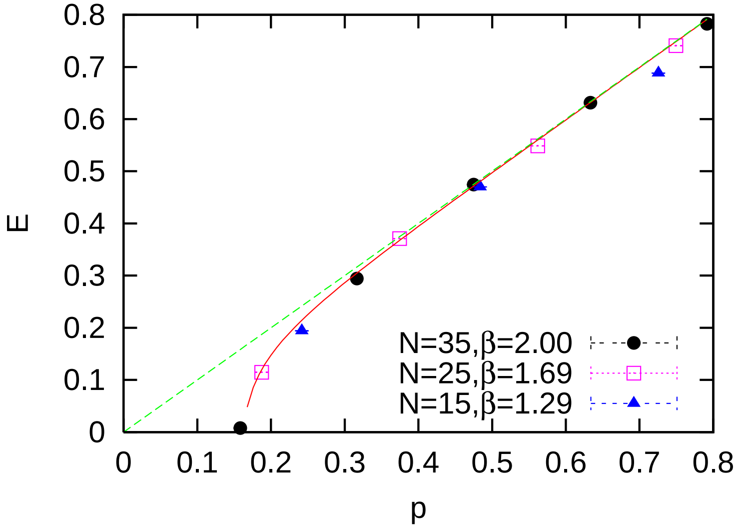
Phase diagram : weak ↔ moderate ↔ strong coupling

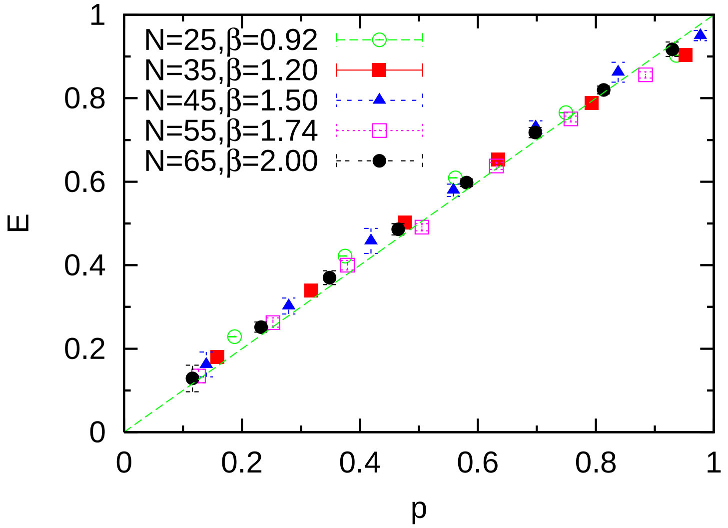


Double Scaling Limit $\beta \propto \sqrt{N}$ always leads to the phase of broken symmetry.

That phase could describe a stable cont. limit for the NC photon.

Dispersion relation: determined from exp. decay in comm. plane $E(p = p_3)|_{p_1=p_2=0}$





sym. phase	**broken phase**
consistent with neg. IR divergence	IR stable
“tachyonic″ behaviour	Goldstone boson

Photon may survive in an NC world,
but explicit prediction for the deformed dispersion relation is outstanding.

- Return to a pragmatic ansatz: J. Ellis et al. (2006/7)

$$\begin{aligned} c^2|\vec{p}|^2 &= E^2\Big(1 + \frac{E}{M}\Big) \\ \rightarrow \quad v_\gamma(E) &= \frac{\partial E}{\partial|\vec{p}|} \simeq c\Big(1 - \frac{E}{M}\Big) \end{aligned}$$

M : very heavy mass, emerges *somehow* from “quantum gravity foam”, noticeable at high energy, *or after a long path.*

Analysis of 35 GRB’s

Data from 3 satellites [e.g. HETE: $dt = 64\ \mathrm{ms}$, 4 energy channels].

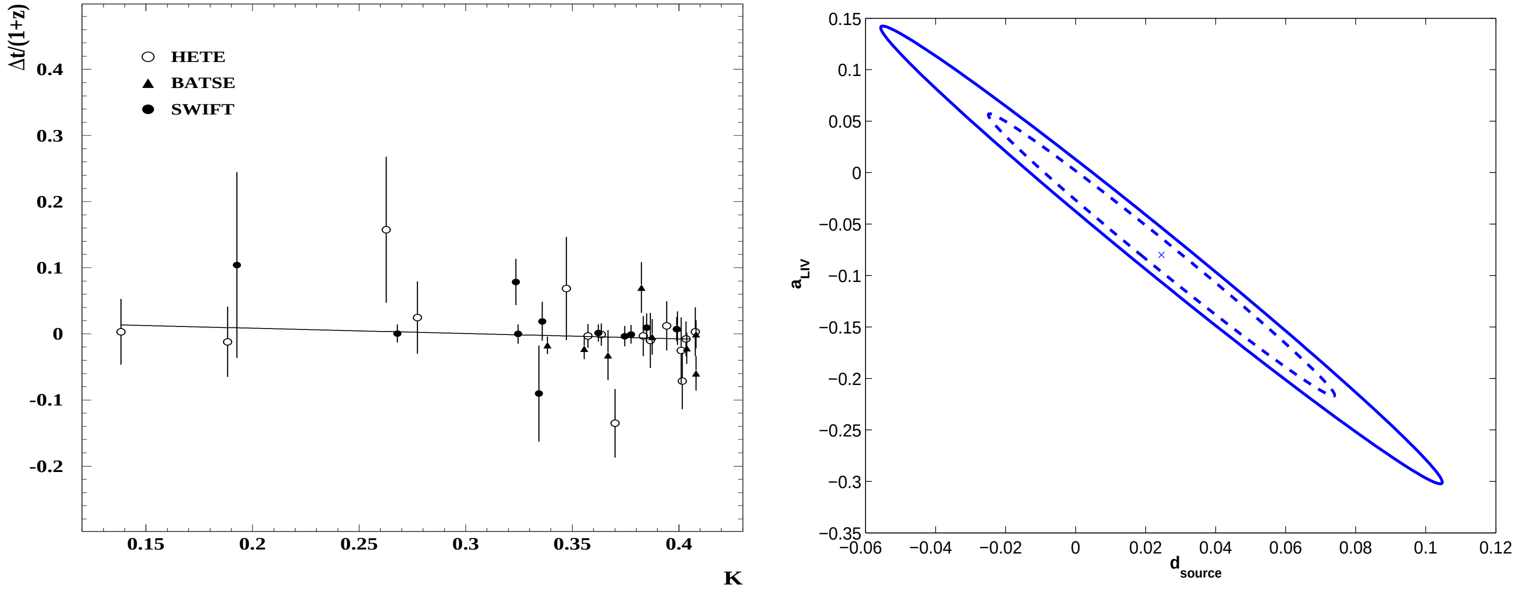
High energy γ’s arrive later. Ansatz for the observed delay *without LIV*

$$\Delta t_{\mathrm{obs}} = d_{\mathrm{source}}(1+z)$$

d_{source} : possible delay already in the emission
z : redshift

With LIV: $$\frac{\Delta t_{\mathrm{obs}}}{1+z} = d_{\mathrm{source}} + \underbrace{\frac{\Delta E}{M}\frac{1}{H_0}}_{a_{\mathrm{LIV}}} K(z) \qquad (K \;:\; \text{complicated correction})$$

Enhance error bars until fits match: 1 σ evidence for LIV [68 %, **95 %**]



Cautious conclusion : $\underline{|\mathrm{M}| > 1.4 \cdot 10^{25}\ \mathrm{eV} \approx 0.001\ \mathrm{M}_{\mathrm{Planck}}}$ (with 95 % C.L.)

Studies of single GRBs or blazar flares ($e.g.$ Mkn501, Mkn 421) even conclude $|M| > 0.01 M_{\mathrm{Planck}}$

Conclusions:

Cosmic rays: unique opportunity for phenomenological access to tremendous energies.

In the centre-of-mass frame, relevant processes are harmless low energy events
$\rightarrow$ Question of LI is crucial !

GZK and γ-TeV Puzzle :

Why is the Universe surprisingly transparent for
• protons with $E_p \gtrsim 10^{20}$ eV • photons with $E_\gamma \gtrsim 10$ TeV ?

Open question; LIV provides a class of proposals for a solution, *IF some puzzle persists*

LIV not detected anywhere (except OPERA) — we discussed failed attempts with cosmic neutrinos and GRBs. But established LI precision does not exclude proposed solutions.

New projects include : Japanese Experiment Module – Extreme Universe Space Observatory (JEM-EUSO), Orbiting Wide-angle Light-collectors (OWL): search for fluorescence light from satellites Pierre Auger: new plant in northern hemisphere . . .

News in Nov. 2007 : **AGN Hypothesis**

[Active Galactic Nuclei: in centre a super-massive black hole ($> 10^6$ solar masses), nucleus attracts and absorbs large quantities of matter, but emits high-E particles (mechanism ?)]

Pierre Auger Collab. analyses UHECRs detected from Jan. 2004 - May 2006

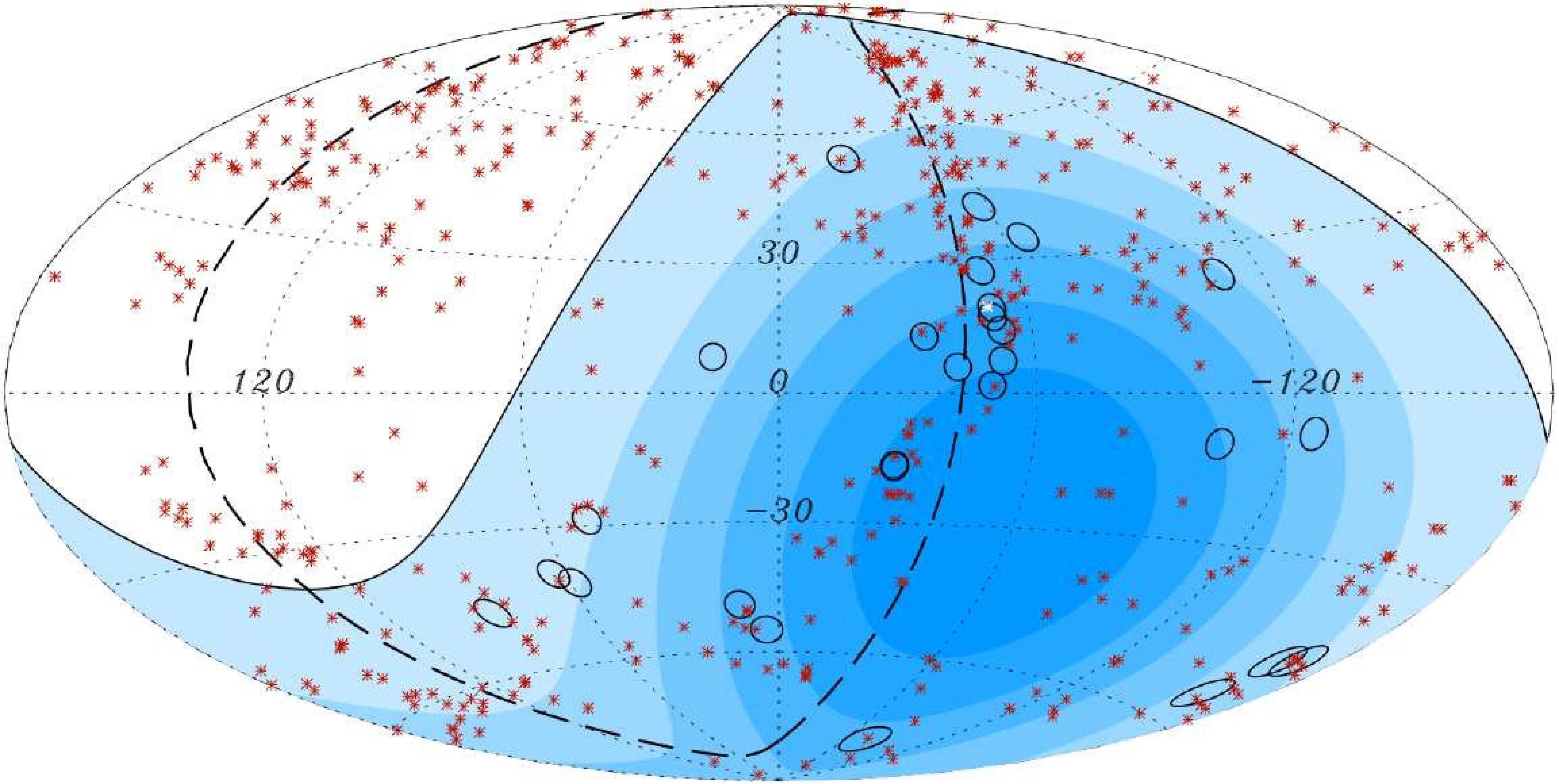
Hypothesis: directions are clustered and correlated with locations of nearby AGN

3 parameters:
ψ : angular range around UHECR direction
$E_{\rm min}$: threshold for UHECR
$R_{\rm max}$: max. distance to "nearby" AGN (from redshift)

tuning $\to$ $(\psi, E_{\rm min}, R_{\rm max}) = (3.1^0,\ 5.6 \cdot 10^{19}\ {\rm eV},\ 75\ {\rm Mpc})$
yields max. correlation, captures 12 our of 15 UHECR
(for isotropic sources: 3.2 expected *[at fixed parameters . . .]*)

$R_{\rm max}$ short, $\approx$ straight UHECR propagation conceivable

Check with data from May 2006 - Aug. 2007: captures 8 out of 13 UHECRs (2.8 expected)



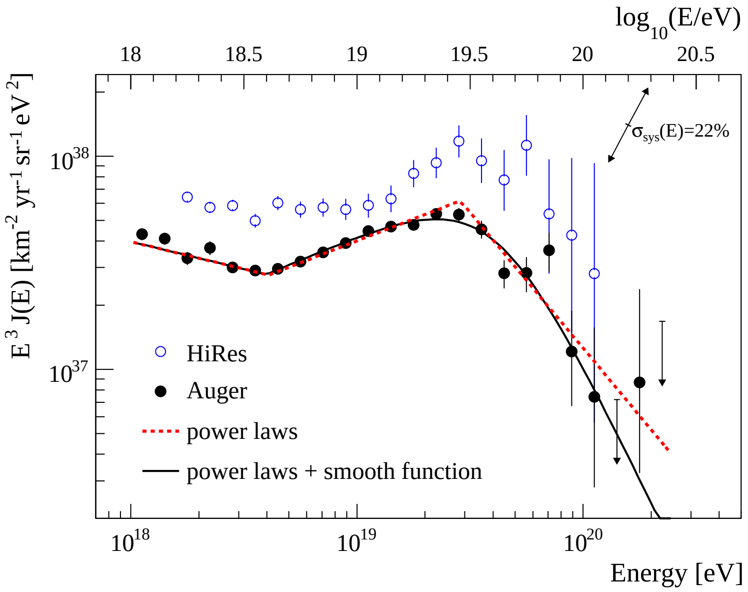
Celestial sphere with circles of radius 3.1° at arrival directions of 27 UHECRs detected by the Pierre Auger Observatory. Asterisks: 472 AGN with $R < 75$ Mpc.
Dashed : supergalactic plane; white : Centaurus A

For **clustering** *PA Collab. claims 99 % C.L.,* **correlation** *less clear*

Critics :

- Variation of $(\psi, E_{\min}, R_{\max})$ is discussed only vaguely.
- Statistics still small, world data before essentially isotropic
  [AGASA (1996): slight signal for clustering, contradicted by HiRes]
  Consistent with world data ?
- Gorbunov/Tinyakov/Tkachev/Troisky :
  Flux $\propto 1/R^2 \;\rightarrow$ nearest AGN should be dominant sources
  in particular Cen A and Virgo should contribute each ≈ 6 events out of 27
  Cen A in business, but Virgo delivers none
  $\Rightarrow$ AGN Hypothesis disfavoured at 99 % C.L.
  [However:
  argument could be evaded if AGN are *episodic* UHECR sources]
- D. Fargion: short $R_{\max}$ favours heavy nuclei as primaries
- Hypothesis *supported* by new Irkutsk data analysis, but *not* by HiRes; AGASA pending.

Updated spectrum: Pierre Auger Collaboration (2010)



flux $(E \lesssim E_{\text{ankle}} \simeq 10^{18.6}\ \text{eV}) \propto E^{-3.3}$ (stat. errors with Feldman/Cousins method)

flux $(E_{\text{ankle}} < E < E_{\text{GZK}} \simeq 10^{19.6}\ \text{eV}) \propto E^{-2.6}$

Just beyond : clearly suppressed, but in good agreement with $E^{-3.3}$ extrapolation

GZK cutoff is substantiated ?

Clustering and AGN Hypotheses

period	exposure [km^2 sr yr]	events above E_{min}	AGN direction	isotropically expected
until May 2006	4390	14	9	2.9
June 2006 - Aug. 2007	4500	13	9	2.7
Sept. 2007 - March 2009	8150	31	8	6.5

1. period: exploratory, used to fix paramters (→ biased)

E_{min} shifted down to $5.5 \cdot 10^{19}$ eV (calibration corrected)

Critics addressed: Virgo passivity persisits, but only 1.2 events expected
"masked data" excluding vicinity of galactic plane (12°): no drastic change

New data with exposure almost doubled: "neither strenghten nor contradict" hypotheses of clustering and ANG correlation. Overall still supported (in particular clustering), but evidence became clearly weaker.

Probability for accidental isotropic effect: $p = 0.0004$ (2006/7), $p = 0.33$ (2007/9).