Contact Hamiltonian Dynamics

Alessandro Bravetti

alessandro.bravetti@cimat.mx

CIMAT-Guanajuato-Mexico

CIMAT

Seminario de Partículas, Campos y Relatividad General
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Outline of the seminar

1. Motivation
2. Contact Manifolds
3. Contact Hamiltonian Dynamics
4. Applications
5. Further subjects

Motivation

Motivation 1: Geometry

Quote (Arnold)

"The relations between symplectic and contact geometries are similar to those between linear algebra and projective geometry. First, the two related theories are formally more or less equivalent: every theorem in symplectic geometry may be formulated as a contact geometry theorem, and any assertion in contact geometry may be translated into the language of symplectic geometry. Next, all the calculations look algebraically simpler in the symplectic case, but geometrically things are usually better understood when translated into the language of contact geometry. Hence one is advised to calculate symplectically but to think rather in contact geometry terms"

Goal

Introduce contact geometry and contact Hamiltonian systems

Motivation 2: Physics

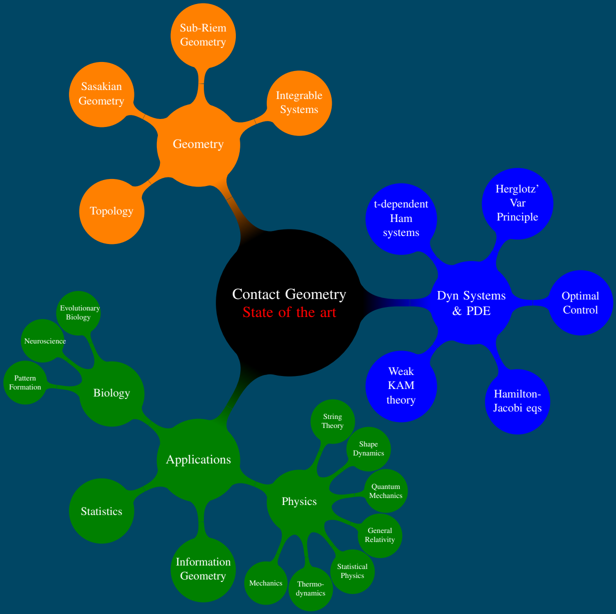
Quote (Arnold)

"Every mathematician knows it is impossible to understand an elementary course in thermodynamics. The reason is that thermodynamics is based–as Gibbs has explicitly proclaimed–on a rather complicated mathematical theory, on the contact geometry. Contact geometry is one of the few simple geometries of the so-called Cartan's list, but it is still mostly unknown to the physicist–unlike the Riemannian geometry and the symplectic or Poisson geometries, whose fundamental role in physics is today generally accepted"

Goal

Introduce contact geometry and contact Hamiltonian systems and their relevance in physics

Motivation 3: Geom., Phys. & More



Contact Manifolds

Symplectic Vs Contact

Definitions: Nice Vs Ugly

Symplectic Manifold

$$(M^{2n}, \Omega),$$
$$\mathrm{d}\Omega = 0,\ V \equiv \Omega^n \neq 0$$

(General) Contact Manifold

$$(\mathcal{T}^{2n+1}, \mathcal{D}),$$
"$\mathcal{D}$ max. non-int."

Definition

A contact manifold is a $(2n+1)$-dimensional manifold $\mathcal{T}$, endowed with a contact structure, that is, a maximally non-integrable distribution $\mathcal{D} \subset T\mathcal{T}$ of hyperplanes

Symplectic Vs Contact

Definitions: Nice Vs Nice

Symplectic Manifold

$$(M^{2n}, \Omega),$$
$$\mathrm{d}\Omega = 0,\ V \equiv \Omega^n \neq 0$$

Theorem (Darboux)

In the neighborhood of any point on a symplectic manifold, it is always possible to find a set of local coordinates such that the 2-form Ω can be written

$$\Omega = \mathrm{d}p_a \wedge \mathrm{d}q^a$$

(Exact) Contact Manifold

$$(\mathcal{T}^{2n+1}, \mathcal{D} = \ker(\eta)),$$
$$V \equiv \eta \wedge (\mathrm{d}\eta)^n \neq 0$$

Theorem (Darboux)

In the neighborhood of any point on a contact manifold, it is always possible to find a set of local coordinates such that the 1-form η can be written

$$\eta = \mathrm{d}w - p_a \mathrm{d}q^a$$

Symplectic Vs Contact
Examples

Symplectic Manifold

$$(M^{2n}, \Omega),\ \mathrm{d}\Omega = 0,\ V \equiv \Omega^n \neq 0$$

Canonical coordinates:

$$(q, p) \qquad \Omega = \mathrm{d}p_a \wedge \mathrm{d}q^a$$

Examples:

1. $\mathbb{R}^{2n}$ + standard symplectic
   $$(\mathbb{R}^{2n}, \Omega),\ \Omega = \mathrm{d}p_a \wedge \mathrm{d}q^a$$
2. Cotangent bundle
   $$(T^*M, \Omega)$$
   $$\alpha = p_a \mathrm{d}q^a, \quad \Omega = \mathrm{d}\alpha$$

Contact Manifold

$$(\mathcal{T}^{2n+1}, \eta),\ V \equiv \eta \wedge (\mathrm{d}\eta)^n \neq 0$$

Contact coordinates:

$$(q, p, w) \qquad \eta = \mathrm{d}w - p_a \mathrm{d}q^a$$

Examples:

1. $\mathbb{R}^{2n+1}$ + standard contact
   $$(\mathbb{R}^{2n+1}, \eta),\ \eta = \mathrm{d}w - p_a \mathrm{d}q^a$$
2. 1-jet bundle
   $$(J^1M, \eta) = (T^*M \times \mathbb{R}, \eta)$$
   $$\eta = \mathrm{d}w - \alpha$$

Symplectic Vs Contact
Reeb Vector Field

Symplectic Manifold

$$(M^{2n}, \Omega),\ \mathrm{d}\Omega = 0,\ V \equiv \Omega^n \neq 0$$

Canonical coordinates:

$$(q, p) \qquad \Omega = \mathrm{d}p_a \wedge \mathrm{d}q^a$$

Contact Manifold

$$(\mathcal{T}^{2n+1}, \eta),\ V \equiv \eta \wedge (\mathrm{d}\eta)^n \neq 0$$

Contact coordinates:

$$(q, p, w) \qquad \eta = \mathrm{d}w - p_a \mathrm{d}q^a$$

Reeb (Characteristic) vector field:

$$\mathrm{d}\eta(\xi, \cdot) = 0 \qquad \eta(\xi) = 1$$

In contact coordinates:

$$\xi = \frac{\partial}{\partial w}$$

Symplectic Vs Contact
Symmetries

Symplectic Manifold

$$(M^{2n}, \Omega),\ \mathrm{d}\Omega = 0,\ V \equiv \Omega^n \neq 0$$

Canonical coordinates:

$$(q, p) \qquad \Omega = \mathrm{d}p_a \wedge \mathrm{d}q^a$$

Canonical transformations:

$$\begin{aligned} \tilde{\Omega} &= \mathrm{d}\tilde{p}_a \wedge \mathrm{d}\tilde{q}^a \\ &= \mathrm{d}p_a \wedge \mathrm{d}q^a \\ &= \Omega \end{aligned}$$

Contact Manifold

$$(\mathcal{T}^{2n+1}, \eta),\ V \equiv \eta \wedge (\mathrm{d}\eta)^n \neq 0$$

Contact coordinates:

$$(q, p, w) \qquad \eta = \mathrm{d}w - p_a \mathrm{d}q^a$$

Contact transformations:

$$\begin{aligned} \tilde{\eta} &= \mathrm{d}\tilde{w} - \tilde{p}_a \mathrm{d}\tilde{q}^a \\ &= f\,(\mathrm{d}w - p_a \mathrm{d}q^a) \\ &= f\,\eta \end{aligned}$$

Contact Symmetries

Example: the Legendre transformation

Definition

Consider a disjoint partition $I \cup J$ of the set of indices $\{1, \ldots, n\}$. A Legendre transformation on $\mathcal{T}$ is given by the relations

$$\begin{aligned} \tilde{w} &:= w - p_I \, q^I \\ \tilde{p}_I &:= q^I \,, \\ \tilde{q}^I &:= -p_I \,, \end{aligned}$$

while leaving the rest of the coordinates unchanged, i.e. $\tilde{q}^J = q^J$, $\tilde{p}_J = p_J$.
When $I \subset \{1, \ldots, n\}$, it is a partial Legendre transformation (PLT).
When $I = \{1, \ldots, n\}$, it is a total Legendre transformation (TLT).

Remark

$$\mathrm{d}\tilde{w} - \tilde{p}_a \mathrm{d}\tilde{q}^a = \mathrm{d}w - p_a \mathrm{d}q^a$$

Symplectic Vs Contact
Submanifolds: Definitions

Lagrange Submanifold

$$N^n \subset M^{2n} \quad \text{s.t.} \quad \Omega|_{TN^n} = 0$$

Legendre Submanifold

$$\mathcal{L}^n \subset \mathcal{T}^{2n+1} \quad \text{s.t.} \quad \eta|_{T\mathcal{L}^n} = 0$$

Theorem (Local characterization)

Consider a disjoint partition $I \cup J$ of the set of indices $\{1, \ldots, n\}$ and a function of n variables $f(p_i, q^j)$, with $i \in I$ and $j \in J$.
The $n+1$ equations

$$q^i = -\frac{\partial f}{\partial p_i} \qquad p_j = \frac{\partial f}{\partial q^j} \qquad w = f - p_i \frac{\partial f}{\partial p_i}$$

define a Legendre submanifold $\mathcal{L}^n$ of $(\mathcal{T}, \eta)$. Conversely, any Legendre submanifold is locally defined by these equations for at least one of the 2^n possible choices of the partition of the set $\{1, \ldots, n\}$.

Symplectic Vs Contact

Submanifolds: Examples

Lagrange Submanifold

$$N^n \subset M^{2n} \quad \text{s.t.} \quad \Omega|_{TN^n} = 0$$

Examples:

1. $\mathbb{R}^n$ in $(\mathbb{R}^{2n}, \mathrm{d}p_a \wedge \mathrm{d}q^a)$
$$N = \{q^a, p_a = c_a\}$$
2. Zero section of T^*M
$$M_0 = \{(q^a, p_a) \mid p_a = 0 \text{ in } T^*_q M\}$$
3. Image of $\mathrm{d}f$ in T^*M
$$N_f = \{(q^a, p_a) \mid p_a = \partial_{q^a} f \text{ in } T^*_q M\}$$

Legendre Submanifold

$$\mathcal{L}^n \subset \mathcal{T}^{2n+1} \quad \text{s.t.} \quad \eta|_{T\mathcal{L}^n} = 0$$

Examples:

1. 1-graph of $f(q^a)$ in J^1M
$$\mathcal{L}_f = \{q^a, p_a = \partial_{q^a} f, w = f\}$$
2. $\mathrm{TLT}(\mathcal{L}_f)$
In general $\mathrm{TLT}(\mathcal{L}_f) \neq \mathcal{L}_f$.
If $f(q^a)$ is convex, then
$$\mathrm{TLT}(\mathcal{L}_f) = \mathcal{L}_{\tilde{f}},$$
where
$$\tilde{f}(p_a) := \max_{q^a}[f(q^b) - p_a q^a]$$

Contact Hamiltonian Dynamics

Symplectic Vs Contact

Dynamics: Definition

Hamiltonian:

$$H : M^{2n} \to \mathbb{R}$$

Dynamics:

$$\Omega(X_H, \cdot) = -\mathrm{d}H$$

Hamiltonian:

$$h : \mathcal{T}^{2n+1} \to \mathbb{R}$$

Dynamics:

$$\eta(X_h) = -h \qquad \pounds_{X_h}\eta = f_h\,\eta$$

Using Cartan:

$$-\xi(h)\,\eta - \mathrm{d}\eta(X_h, \cdot) = -\mathrm{d}h$$

Symplectic Vs Contact
Dynamics: Hamilton's eqs

Hamiltonian:

$$H : M^{2n} \to \mathbb{R}$$

Dynamics:

$$\Omega(X_H, \cdot) = -\mathrm{d}H$$

Hamilton's eqs:

$$\begin{aligned} \dot{q}^i &= \frac{\partial H}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial H}{\partial q^i} \end{aligned}$$

Hamiltonian:

$$h : \mathcal{T}^{2n+1} \to \mathbb{R}$$

Dynamics:

$$-\xi(h)\,\eta - \mathrm{d}\eta(X_h, \cdot) = -\mathrm{d}h$$

Hamilton's eqs:

$$\begin{aligned} \dot{q}^i &= \frac{\partial h}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial h}{\partial q^i} - p_i \frac{\partial h}{\partial w} \\ \dot{w} &= \frac{\partial h}{\partial p_a} p_a - h \end{aligned}$$

Symplectic Vs Contact
Dynamics: Example

Hamiltonian:

$$H = \frac{p^2}{2} + V(q)$$

Dynamics:

$$\Omega(X_H, \cdot) = -\mathrm{d}H$$

Hamilton's eqs:

$$\begin{aligned} \dot{q}^i &= p_i \\ \dot{p}_i &= -\frac{\partial V}{\partial q^i} \end{aligned}$$

Hamiltonian:

$$h = \frac{p^2}{2} + V(q) + \gamma\, w$$

Dynamics:

$$-\xi(h)\,\eta - \mathrm{d}\eta(X_h, \cdot) = -\mathrm{d}h$$

Hamilton's eqs:

$$\begin{aligned} \dot{q}^i &= p_i \\ \dot{p}_i &= -\frac{\partial V}{\partial q^i} - \gamma\, p_i \\ \dot{w} &= \frac{p^2}{2} - V(q) - \gamma\, w \end{aligned}$$

Symplectic Vs Contact

Liouville Theorem

H is conserved:

$$\dot{H} = X_H H = 0$$

Canonical transformations:

$$\pounds_{X_H} \Omega = 0$$

Liouville Theorem:

$$\pounds_{X_H} \Omega^n = 0$$

h is NOT conserved:

$$\dot{h} = X_h h = -\frac{\partial h}{\partial w}\, h$$

Contact transformations:

$$\pounds_{X_h} \eta = -\frac{\partial h}{\partial w}\, \eta$$

Contact Liouville Theorem:

$$\pounds_{X_h} \left(|h|^{-(n+1)} \eta \wedge (\mathrm{d}\eta)^n \right) = 0$$

[Bravetti & Tapias, JPA 48, 245001, (2015)]
[Bravetti & Tapias, PhysRevE 93, 022139, (2016)]

Symplectic Vs Contact
Variational Principles

Lagrange-Hamilton (∼1800):

$$s = \int_0^T L(q, \dot{q})\mathrm{d}t \to \text{extr.}$$

E-L equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}^i}\right) - \frac{\partial L}{\partial q^i} = 0$$

Noether theorem:

✓

Herglotz (∼1930):

$$\dot{w} = L(q, \dot{q}, w) \to \text{extr.}$$

Generalized E-L equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}^i}\right) - \frac{\partial L}{\partial q^i} - \frac{\partial L}{\partial w}\frac{\partial L}{\partial \dot{q}^i} = 0$$

Generalized Noether Theorem:

✓

[Georgieva, Guenther, & Bodurov, JMP 44(9), 3911-3927, (2003)]
[Georgieva, Proceedings of the 12th ICGIQ (2011)]

Symplectic Vs Contact
Hamilton–Jacobi

Stationary:

$$H\left(q, \partial_q s\right) = c$$

Evolutionary:

$$H\left(q, \partial_q s\right) = \frac{\partial s}{\partial t}$$

Characteristic eqs:

$$\begin{aligned} \dot{q}^i &= \frac{\partial H}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial H}{\partial q^i} \end{aligned}$$

Stationary:

$$h\left(q, \partial_q w, w\right) = c$$

Evolutionary:

$$h\left(q, \partial_q w, w\right) = \frac{\partial w}{\partial t}$$

Characteristic eqs:

$$\begin{aligned} \dot{q}^i &= \frac{\partial h}{\partial p_i} \\ \dot{p}_i &= -\frac{\partial h}{\partial q^i} - p_i \frac{\partial h}{\partial w} \\ \dot{w} &= \frac{\partial h}{\partial p_a} p_a - h \end{aligned}$$

Symplectic Vs Contact

Hamilton–Jacobi

Stationary:

$$H(q, \partial_q s) = c$$

Evolutionary:

$$H(q, \partial_q s) = \frac{\partial s}{\partial t}$$

Stationary:

$$h(q, \partial_q w, w) = c$$

Evolutionary:

$$h(q, \partial_q w, w) = \frac{\partial w}{\partial t}$$

[Wang, Wang & Yan, Nonlinearity, 30(2):492, (2016)]
[Wang, Wang & Yan, arXiv:1801.05612, (2018)]

Symplectic Vs Contact
Algebraic Structures

Poisson bracket:

$$\{F,G\} := \Omega(X_F, X_G)$$

In coords:

$$\{F,G\} = \frac{\partial F}{\partial q^a}\frac{\partial G}{\partial p_a} - \frac{\partial G}{\partial q^a}\frac{\partial F}{\partial p_a}$$

Then:

Integrable Systems

Heisenberg Algebra

Jacobi bracket:

$$\{f,g\} := \eta([X_f, X_g])$$

In coords:

$$\{f,g\} = \frac{\partial f}{\partial q^a}\frac{\partial g}{\partial p_a} - \frac{\partial g}{\partial q^a}\frac{\partial f}{\partial p_a} + p_a\left(\frac{\partial f}{\partial w}\frac{\partial g}{\partial p_a} - \frac{\partial g}{\partial w}\frac{\partial f}{\partial p_a}\right) + f\frac{\partial g}{\partial w} - g\frac{\partial f}{\partial w}$$

Then:

Contact Integrable Systems

Contact Heisenberg Algebra

Symplectic Vs Contact
Algebraic Structures

Poisson bracket:

$$\{F, G\} := \Omega(X_F, X_G)$$

In coords:

$$\{F, G\} = \frac{\partial F}{\partial q^a}\frac{\partial G}{\partial p_a} - \frac{\partial G}{\partial q^a}\frac{\partial F}{\partial p_a}$$

Jacobi bracket:

$$\{f, g\} := \eta([X_f, X_g])$$

In coords:

$$\{f, g\} = \frac{\partial f}{\partial q^a}\frac{\partial g}{\partial p_a} - \frac{\partial g}{\partial q^a}\frac{\partial f}{\partial p_a} + p_a\left(\frac{\partial f}{\partial w}\frac{\partial g}{\partial p_a} - \frac{\partial g}{\partial w}\frac{\partial f}{\partial p_a}\right) + f\frac{\partial g}{\partial w} - g\frac{\partial f}{\partial w}$$

[Arnold & Novikov, *Dynamical Systems IV*, Springer, (2001)]
[Bravetti, Chung & Tapias, JPA 50(10):105203, (2017)]

Contact Hamiltonian dynamics.

Applications

Symplectic Vs Contact
Applications

Mechanics
(of conservative systems)

Equilibrium StatMech
(microcanonical measure)

Optimal Control
(Pontryagin Min Principle)

Mechanics
(of dissipative systems)

Equilibrium StatMech
(any target measure)

Optimal Control
(PMP in Thermodynamics)

Contact Hamiltonian Mechanics

$$h = H(q,p) + f(q,p,w)$$

$$\begin{aligned}
\dot{q}^i &= \frac{\partial H}{\partial p_i} + \frac{\partial f}{\partial p_i} \\
\dot{p}_i &= -\frac{\partial H}{\partial q^i} - \frac{\partial f}{\partial q^i} - p_i \frac{\partial f}{\partial w} \\
\dot{w} &= p_a \frac{\partial H}{\partial p_a} - H + p_a \frac{\partial f}{\partial p_a} - f
\end{aligned}$$

Moral: we can obtain different dissipative systems

[Bravetti, Cruz & Tapias, AnnPhys 376 (2017) 1739]

Contact Hamiltonian Mechanics

Example

$$h = \frac{1}{2} p^2 + V(q) + \gamma\, w$$

$$\begin{aligned} \dot{q} &= p \\ \dot{p} &= -\frac{\partial V}{\partial q} - \gamma\, p \\ \dot{w} &= \frac{1}{2} p^2 - V(q) - \gamma\, w \end{aligned}$$

Moral: mechanical systems with linear dissipation

[Bravetti, Cruz & Tapias, AnnPhys 376 (2017) 1739]

Contact Hamiltonian StatMech

Remind: Contact Liouville Theorem

$$\mathrm{d}\mu_h = |h|^{-(n+1)}\eta \wedge (\mathrm{d}\eta)^n$$

Choose:

$$h = [\rho(q,p)\rho(w)]^{-1/(n+1)}$$

Obtain the desired invariant measure:

$$\mathrm{d}\mu_h = \rho(q,p)\rho(w)\,\mathrm{d}p\,\mathrm{d}q\,\mathrm{d}w$$

Moral: equilibrium StatMech, sampling distr.

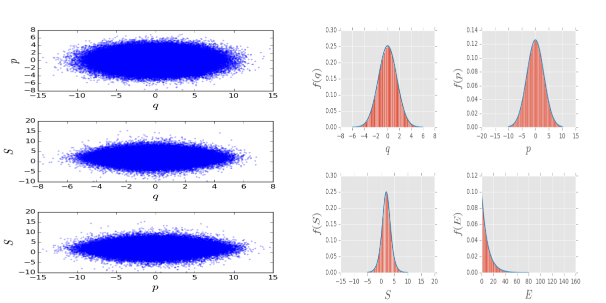
[Bravetti & Tapias, PhysRevE 93, 022139, (2016)]

Contact Hamiltonian StatMech

Example

$$h = \left[e^{-\beta H(q,p)} \rho(w) \right]^{-1/(n+1)}$$

$$d\mu_{\mathrm{sys}} = e^{-\beta H(q,p)} dp\, dq$$



Moral: dynamics for the canonical ensemble

[Bravetti & Tapias, PhysRevE 93, 022139, (2016)]

Contact Hamiltonian ThDynamics

1st Law of TD:

$$\mathrm{d}w - p_a \mathrm{d}q^a = 0$$

Prop1[TD Phase Space (TPS) is contact]:

$$\left(\mathcal{T}^{2n+1} = \{q, p, w\}, \eta = \mathrm{d}w - p_a \mathrm{d}q^a\right)$$

Prop2[TD systems are Legendre submanifolds]:

$$\mathcal{L}_S = \{q, p = \partial_q S, w = S(q)\}$$

Prop3[Conservation of the equilibrium]:

$$\mathcal{L}_S \text{ invariant} \iff h|_{\mathcal{L}_S} = 0$$

Contact Hamiltonian ThDynamics

Example

Entropy Maximum Principle:

$$\begin{cases} \max\limits_{u\in\mathcal{U}} \int_0^{t_f} \dot{S}\,\mathrm{d}t + S(q(t_f)) \\ \dot{q}^i = F^i\left(q, \partial_q S; u\right) \quad \text{(Balance equations)} \\ q^i(0) = q^i_0 \quad \text{(Initial Conditions)} \end{cases}$$

Then [Pontryagin Min Principle]

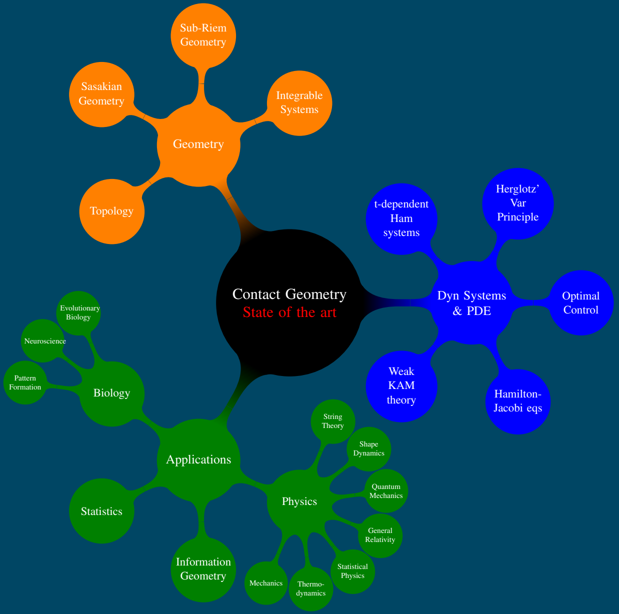
$$\begin{aligned} H(q,p;u) &= \left(p_a - \partial_{q^a} S\right) F^a(q, \partial_q S; u) \\ H(q^*, p^*; u^*) &= \min_{u\in\mathcal{U}} H(q^*, p^*; u) \\ p_i{}^*(t_f) &= \partial_{q^i} S\big|_{q^*(t_f)} \end{aligned}$$

Moral: Thermo-dynamics from Optimal Control

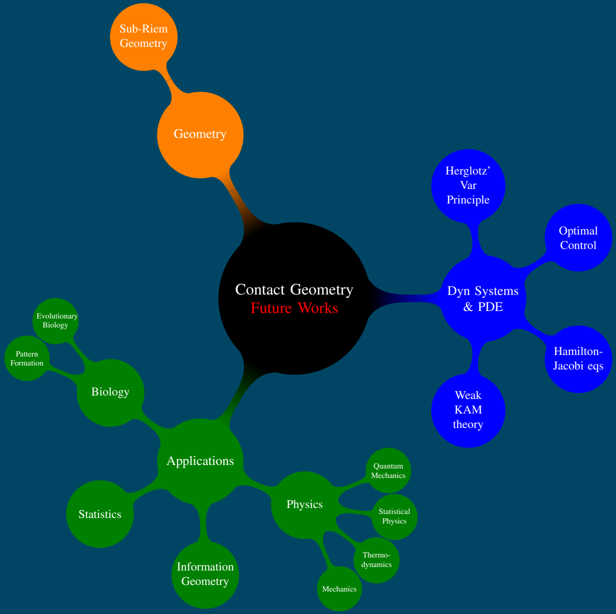
[Bravetti & Padilla, arxiv.org/abs/1804.03309]

Further subjects

Further Subjects



Ongoing & Future Works



Announcements

1. Course on Contact Geometry and TD
   [Bravetti, IJGMMP, 1940003, (2018)]
2. Course on Multi-scale models
   [Modelos Multiescalas: teoría y aplicaciones]
3. Postdoc Positions at CIMAT
   [Convocatoria postdocs 2019]

Thank you!

Contact Gravity

PHYSICAL REVIEW D **95,** 101501(R) (2017)

Action principle for action-dependent Lagrangians toward nonconservative gravity: Accelerating universe without dark energy

Matheus J. Lazo,[1,*] Juilson Paiva,[1] João T. S. Amaral,[1] and Gastão S. F. Frederico[2,3]

[1] *Instituto de Matemática, Estatística e Física—FURG, Rio Grande 96201-900, Rio Grande do Sul, Brazil*

[2] *Departamento de Matemática, Universidade Federal de Santa Catarina, Florianópolis 88040-900, Santa Catarina, Brazil*

[3] *Department of Science and Technology, University of Cape Verde, Praia 7600, Cape Verde*

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In the present work, we propose an action principle for action-dependent Lagrangians by generalizing the Herglotz variational problem for several independent variables. This action principle enables us to formulate Lagrangian densities for nonconservative fields. In particular, from a Lagrangian depending linearly on the action, we obtain generalized Einstein field equations for nonconservative gravity and analyze some consequences of their solutions for cosmology and gravitational waves. We show that the nonconservative part of the field equations depends on a constant cosmological four-vector. Depending on this four-vector, the theory displays damped/amplified gravitational waves and an accelerating Universe without dark energy.

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Contact Quantum Mechanics 1

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Quantization of contact manifolds and thermodynamics

S.G. Rajeev

Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA

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Contact Quantum Mechanics 2

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Contact geometry and quantum mechanics

Gabriel Herczeg [a], Andrew Waldron [b,*]

[a] Department of Physics, University of California, Davis, CA 95616, USA
[b] Center for Quantum Mathematics and Physics (QMAP), Department of Mathematics, University of California, Davis, CA 95616, USA

CONTACT QUANTIZATION: QUANTUM MECHANICS = PARALLEL TRANSPORT

G. HERCZEG♯, E. LATINI♭ & ANDREW WALDRON♮

Abstract.
Quantization together with quantum dynamics can be simultaneously formulated as the problem of finding an appropriate flat connection on a Hilbert bundle over a contact manifold. Contact geometry treats time, generalized positions and momenta as points on an underlying phase-spacetime and reduces classical mechanics to contact topology. *Contact quantization* describes quantum dynamics in terms of parallel trans-

Contact Quantum Mechanics 3

Contact manifolds and dissipation, classical and quantum

F. M. Ciaglia[1,5], H. Cruz[2,6], and G. Marmo[3,4,7]

[1] *Max Planck Institute for Mathematics in the Sciences, Inselstrasse 22, 04103 Leipzig, Germany*

[2] *Institut für Theoretische Physik, JW Goethe-Universität Frankfurt am Main, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany.*

[3] *Dipartimento di Fisica "Ettore Pancini", Università di Napoli "Federico II", Complesso Universitario di Monte S. Angelo, via Cintia, I-80126 Napoli, Italy*

[4] *INFN - Sezione di Napoli, Complesso Universitario di Monte S. Angelo, via Cintia, I-80126 Napoli, Italy*

[5] personal e-mail: `florio.m.ciaglia@gmail.com`
[6] `prado@itp.uni-frankfurt.de`
[7] `marmo@na.infn.it`

Abstract

Motivated by a geometric decomposition of the vector field associated with the Gorini–Kossakowski–Lindblad–Sudarshan (GKLS) equation for finite-level open quantum systems, we propose a generalization of the recently introduced contact Hamiltonian systems for the description of dissipative-like dynamical systems in the context of (non-necessarily exact) contact manifolds. In particular, we show how this class of dynamical systems naturally emerges in the context of Lagrangian Mechanics and in the case of nonlinear evolutions on the space of pure states of a finite-level quantum system.

Contact Shape Dynamics

PHYSICAL REVIEW D **97,** 123541 (2018)

Dynamical similarity

David Sloan[*]

Beecroft Institute of Particle Astrophysics and Cosmology, Department of Physics, University of Oxford, Denys Wilkinson Building, 1 Keble Road, Oxford OX1 3RH, United Kingdom

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We examine "dynamical similarities" in the Lagrangian framework. These are symmetries of an intrinsically determined physical system under which observables remain unaffected, but the extraneous information is changed. We establish three central results in this context: (i) Given a system with such a symmetry there exists a system of invariants which form a subalgebra of phase space, whose evolution is autonomous; (ii) this subalgebra of autonomous observables evolves as a contact system, in which the frictionlike term describes evolution along the direction of similarity; (iii) the contact Hamiltonian and one-form are invariants, and reproduce the dynamics of the invariants. As the subalgebra of invariants is smaller than phase space, dynamics is determined only in terms of this smaller space. We show how to obtain the contact system from the symplectic system, and the embedding which inverts the process. These results are then illustrated in the case of homogeneous Lagrangians, including flat cosmologies minimally coupled to matter; the n-body problem and homogeneous, anisotropic cosmology.

Contact String Theory

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Completely Integrable Contact Hamiltonian Systems and Toric Contact Structures on $S^2 \times S^3$ ⋆

Charles P. BOYER

Department of Mathematics and Statistics, University of New Mexico, Albuquerque, NM 87131, USA
E-mail: *cboyer@math.unm.edu*
URL: `http://www.math.unm.edu/~cboyer/`

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Abstract. I begin by giving a general discussion of completely integrable Hamiltonian systems in the setting of contact geometry. We then pass to the particular case of toric contact structures on the manifold $S^2 \times S^3$. In particular we give a complete solution to the contact equivalence problem for a class of toric contact structures, $Y^{p,q}$, discovered by physicists by showing that $Y^{p,q}$ and $Y^{p',q'}$ are inequivalent as contact structures if and only if $p \neq p'$.

Key words: complete integrability; toric contact geometry; equivalent contact structures; orbifold Hirzebruch surface; contact homology; extremal Sasakian structures

2010 Mathematics Subject Classification: 53D42; 53C25

Contact Monte Carlo

Adiabatic Monte Carlo

Michael Betancourt
Department of Statistics, University of Warwick, Coventry CV4 7AL, UK[*]
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A common strategy for inference in complex models is the relaxation of a simple model into the more complex target model, for example the prior into the posterior in Bayesian inference. Existing approaches that attempt to generate such transformations, however, are sensitive to the pathologies of complex distributions and can be difficult to implement in practice. Leveraging the geometry of equilibrium thermodynamics, I introduce a principled and robust approach to deforming measures that presents a powerful new tool for inference.

Contact and information geometric description of an extended Markov Chain Monte Carlo method

Shin-itiro GOTO[1] and Hideitsu HINO[2],
1)Department of Applied Mathematics and Physics, Graduate School of Informatics,
Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto, 606-8501, Japan
2) The Institute of Statistical Mathematics,
10-3 Midori-cho, Tachikawa, Tokyo 190-8562, Japan

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Contact PDEs

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Symmetries of the Generalized Variational Functional of Herglotz for Several Independent Variables

Bogdana Georgieva

Abstract. This paper provides a method for calculating the symmetry groups of the functional defined by the generalized variational principle of Herglotz in the case of several independent variables. Examples of calculating variational symmetry groups are given, including those for the non-conservative nonlinear Klein-Gordon equation, and for the equations describing the propagation of electromagnetic fields in a conductive medium.

Keywords. Variational symmeties, Herglotz variational principle, invariant functional, Herglotz

Mathematics Subject Classification (2000). 49

Contact Optimal Control

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Contact geometry of the Pontryagin maximum principle☆

Tomoki Ohsawa

Department of Mathematical Sciences, The University of Texas at Dallas, 800 W Campbell Rd, Richardson, TX 75080-3021, United States

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ABSTRACT

This paper gives a brief contact-geometric account of the Pontryagin maximum principle. We show that key notions in the Pontryagin maximum principle — such as the separating hyperplanes, costate, necessary condition, and normal/abnormal minimizers — have natural contact-geometric interpretations. We then exploit the contact-geometric formulation to give a simple derivation of the transversality condition for optimal control with terminal cost.

Contact Biology

SCIENTIFIC REPORTS

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An optimal strategy to solve the Prisoner's Dilemma

Alessandro Bravetti[1] & Pablo Padilla[1,2]

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Cooperation is a central mechanism for evolution. It consists of an individual paying a cost in order to benefit another individual. However, natural selection describes individuals as being selfish and in competition among themselves. Therefore explaining the origin of cooperation within the context of natural selection is a problem that has been puzzling researchers for a long time. In the paradigmatic case of the Prisoner's Dilemma (PD), several schemes for the evolution of cooperation have been proposed. Here we introduce an extension of the Replicator Equation (RE), called the Optimal Replicator Equation (ORE), motivated by the fact that evolution acts not only at the level of individuals of a population, but also among competing populations, and we show that this new model for natural selection directly leads to a simple and natural rule for the emergence of cooperation in the most basic version of the PD. Contrary to common belief, our results reveal that cooperation can emerge among selfish individuals because of selfishness itself: if the final reward for being part of a society is sufficiently appealing, players spontaneously decide to cooperate.